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Essays in Dynamic Macroeconomics

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Essays in Dynamic Macroeconomics

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A Naty, mis viejos, mis hermanas y mis abuelos.

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Essays in Dynamic Macroeconomics

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The focus of my research is dynamic macroeconomics and how the economy responds to changes in government policy. During the last 30 years, the sovereign bond market in emerging economies has grown considerably and many large scale defaults were observed. Existing models of sovereign debt are unable to jointly explain the debt to output ratios and the default frequency in these countries. In the first chapter, to address this puzzle, I propose a standard small open economy model with the addition that the government transits through different political states and these transitions cannot be directly observed by lenders. Moreover, after a default, the government chooses when to renegotiate and it bargains with the lenders over the recovery rate. I show that government reputation and endogenous periods of exclusion and recovery rates play a crucial role in explaining this phenomenon. In the

second chapter, I use a dynamic political economy model to evaluate whether the observed rise in wage inequality and decrease in median to mean wages can explain the increase in transfers to low earnings quintiles and increase in effective tax rates for high earnings quintiles in the U.S. over the past several decades. I conduct a welfare analysis by contrasting the solution from the political mechanism with those from a sequential utilitarian mechanism, as well as mechanisms with commitment. Finally, the third chapter focuses on explaining the dynamics of firms. I ask whether an entry/exit model like that pioneered by Hopenhayn (1992, Econometrica) with a capital accumulation decision and non-convex costs of adjustment can generate size and age dependence like that found in the data. In particular, conditional on age, growth, employment creation and destruction and volatility are decreasing in size. Moreover, conditional on size, growth, employment creation and destruction and volatility are decreasing in age. The main point of this chapter is to demonstrate that a model with no financial frictions parameterized to match the investment regularities of U.S. establishments is able to account for the simultaneous dependence of industry dynamics on size (once we condition on age) and on age (once we condition on size).

To explain how the economy responds and conduct welfare analysis either one has to find natural experiments or one has to build computational models and run counterfactual experiments. My research follows the latter strategy.

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Chapter 1

Government Reputation and Debt Repayment in Emerging Economies

1.1 Introduction

Evidence shows that emerging economies that repeatedly defaulted on their external obligations are still able to accumulate considerable amounts of debt. Table 1.1 displays the average government debt to output ratio for countries that defaulted in the last decade. The average country, among emerging economies, experienced 3 defaults every 100 years and sustains an average external debt to output ratio around 58%.[1] One of the primary goals of a quantitative model of emerging market

[1] Cases considered are defaults and/or debt restructurings (including reschedulings) in distressed circumstances involving external creditors. Defaults triggered by wars, revolutions, occupations and state disintegrations are excluded. All restructurings and defaults refer to federal or central government distress. Previous defaults since 1824 or year of independence (Pakistan 1947, Indonesia 1949, Ukraine 1991, Uruguay 1830, Grenada 1974). The debt to output ratio is computed using quarterly output and the stock of debt corresponds to external public debt with private creditors.

Table 1.1: Government Debt to Output Ratios and Default Episodes

Country	Gov.Debt/GNP 1970-2005 (%)	# of defaults	last default year t	Gov.Debt/GNP (%) year $t-1$
Russia	51	3	1998	80
Pakistan	9	2	1999	13
Ecuador	122	7	1999	200
Ukraine	24	2	2000	41
Argentina	88	5	2001	103
Indonesia	40	3	2002	28
Paraguay	28	7	2003	39
Uruguay	79	6	2003	146
Grenada	41	1	2004	208
Venezuela	98	8	2005	75
Average	58	4		93

Sources: World Bank, *Global Development Finance II*;
Sturzenegger and Zettelmeyer (2005); and Standard & Poor's (2007).

debt should be to generate the observed frequency of default in an equilibrium that sustains a level of external obligations similar to those displayed in Table 1.1. However, this remains a puzzle. Important recent contributions in the sovereign debt literature, such as Arellano (2007), Aguiar and Gopinath (2006) and Yue (2007), are unable to account for both debt to output ratios and recurrent default events.[2,3] In this paper, I ask whether a model of government reputation where the risk of default

[2] Arellano (2007), Aguiar and Gopinath (2006) and Yue (2007) are calibrated to Argentina and predict that, at the observed default frequency, the debt to *quarterly* output ratio should be 7.3% in the work of Arellano (2007); 18% in the case of Aguiar and Gopinath (2006); and 9.7% for Yue (2007).

[3] This was also a challenge in recent studies of unsecured consumer debt such as Chatterjee, Corbae, Nakajima and Rios Rull (2007, Econometrica) because high frequency of default makes unsecured debt very expensive which deters consumer borrowing. In addition to earnings shocks, and in order to jointly match the level of consumer debt and the default frequency, they consider unexpected medical expenses and private life-events (such as divorce) as a possible trigger for default.

and debt renegotiation are endogenous, that accounts for the default frequency, can generate a higher debt to output ratio.

During the last two decades the sovereign debt market has become increasingly competitive and it can be characterized as follows.[4] First, defaults are associated with "bad times" of the economy. Second, defaults are always followed by some form of repayment. Third, sovereign ratings, viewed as a measure of the willingness to pay of the sovereign, play an important role in determining countries access to international credit markets and are highly correlated with countries' interest rates. Fourth, countries do not access capital markets while the default situation lasts.

Based on the previous facts, I develop a model of sovereign debt, default and government reputation where the country is subject to income fluctuations. In this economy, an altruistic government makes the borrowing decisions and there is no commitment technology to repay the debt. Following Cole, Dow and English (1995), the government transits through two political states that affect how the government values the future. The state evolves as a Markov process rather than being permanent. According to these states we can classified the government as "aligned", if the government discounts the future at the same rate as consumers, or "misaligned", if the government discounts the future at a higher rate. The sovereign and risk neutral competitive financial intermediaries trade one period non-contingent bonds. A government default leads to financial autarky. However, the lenders and the government can renegotiate over debt a reduction. Only after repayment of the renegotiated debt can a government regain access to capital markets. Debt recovery rates are determined in a Nash bargaining mechanism consistent with the information structure.

The parameters of the model are estimated via simulated method of moments. In order to make a fair comparison of the results in this paper to those

[4]See Kletzer (1994), Eaton and Fernandez (1995), Sturzenegger and Zettelmeyer (2006a) and Section 1.1.2 below.

in previous studies in the sovereign debt literature, the moments chosen for the estimation are similar to the moments used in the calibration of Arellano (2007), Aguiar and Gopinath (2006) and Yue (2007).[5] It is important to note that the set of targeted moments does not include the debt to output ratio. After the estimation is done, and as a test of the model, I ask whether the mean debt to output ratio is consistent with the values presented in Table 1.1. I show that, at the observed default frequency, the combination of government reputation and endogenous recovery rates generates a debt to output ratio around 40% that is between 2 to 5 times higher than previous models in the literature.

One of the contributions of the paper is to develop a framework that is consistent with how the sovereign debt market for emerging economies works. International lenders learn about the political state from government borrowing and repayment behavior and summarize its reputation in the probability of a government being of the aligned type. I show that we can identify this probability with the sovereign rating. In equilibrium, the aligned government (hereafter the $a - type$), has a lower probability of default and borrow less than the misaligned government (hereafter the $m - type$). Thus, a default decision or taking more debt lowers the sovereign rating. Moreover, the $a - type$ government has a higher probability of entering the renegotiation stage than the $m - type$ government. This implies that after repayment the model generates an upgrade in the sovereign rating. Reputation becomes valuable because the terms of international loans depend not only on economic fundamentals, but also on the sovereign rating.

Another important contribution of this paper is that incorporates both sovereign default and debt renegotiation into a dynamic general equilibrium model with private information. Post-default renegotiation and the endogenous determination of exclusion periods have important effects over the incentives to default. However,

[5] As in these papers, I also base my analysis on the sovereign debt of Argentina.

most of the recent papers in the literature, except Yue (2007), have considered zero recovery rates and exogenous exclusion periods after a default. Compared to a world with zero recovery rates (the environment of Arellano (2007) and Aguiar and Gopinath (2006)), interest rates at a particular debt level are lower because lenders have an expected recovery value that might be different from zero. Second, endogenous recovery rates introduce a level of contingency to debt contracts (they are function of the state of the economy in the repayment period) and make borrowing more appealing. As opposed to Yue (2007), countries are not forced to renegotiate in the default period and recovery rates are determined in the repayment period. Consistent with the data, countries have stronger incentives to repay in good times (see Kovrijnykh and Szentes (2007)).

The information structure and the endogenous determination of default penalties (recovery rates and periods of exclusion) are essential ingredients to obtain the main result. In section 1.6, I show that a model with full information and zero recovery rates generates an equilibrium debt to output ratio that is only 34% of that generated by the benchmark model. In addition, a model with full information and endogenous recovery generates a debt to output ratio that is only 72% of the ratio in the model with private information. In these models, reputation has no value (political states are observable), thus a repayment decision per se has no effect on future prices.

Emerging economies are characterized by volatile business cycles. Interest rates are countercyclical and highly volatile. Moreover, the current account is countercyclical and positively correlated with interest rates. At the estimated parameters, the benchmark model also accounts well for the business cycle behavior in these countries.

1.1.1 Related Literature

The pioneering work on sovereign debt and reputation is Eaton and Gersovitz (1981). In their model, countries were permanently excluded from credit markets after a default, so country's incentive to make repayments is to preserve its future access to foreign lenders. Bulow and Rogoff (1989) state that if markets are complete, sovereign debt can be sustained in equilibrium, only if it is possible to impose sanctions over a country after a default. I choose to model debt as a one period uncontingent bonds and countries are not allowed to borrow or save during default periods so their argument does not apply in this context. Moreover, the empirical evidence rejects the imposition of sanctions as a possible explanation of the existence of debt with no commitment.

This paper is closely related with Cole, Dow and English (1995) and Phelan (2006). They also study models with heterogenous agents where a players' type changes over time and is private information. Cole, et. al. (1995) focused on nineteenth-century bond defaults and subsequent resumptions. The level of debt and the repayment amount after a default are fixed. Phelan (2006) studies the effects of government reputation and shows that the unique equilibrium has the opportunistic government following a mixed strategy. Chatterjee, Corbae and Rios Rull (2007) also consider an environment with heterogenous borrowers and private information. They focus on unsecured consumer debt and the welfare consequences of imposing legal restrictions on the length of time that adverse events can remain on individual's credit record. The credit scoring technology implemented in this paper is similar to theirs. Another related paper is Alfaro and Kanczuk (2005). They study a model of sovereign debt with adverse selection where governments differ in their patience level but the borrowing level is exogenously given and countries have continuous access to credit markets. Moreover, they study equilibria where the government with a lower discount rate always default.

Recent quantitative models have related business cycles and sovereign debt in environments where the government cannot commit to pay back. Besides the models of Arellano (2007), Aguiar and Gopinath (2006) and Yue (2007) referenced before, other papers employed similar environments. In particular, Hatchondo, Martinez and Sapriza (2007) consider an environment where the government type can change but they assume zero debt recoveries and no exclusion from credit markets after a default. In their paper, government types are public information and debt contracts are contingent on types similar to the full information specification of the benchmark model. Amador (2003) and Cuadra and Sapriza (2006) study sovereign default in a setup in which different government types alternate in power. The types disagree on the optimal allocation of resources within each period but do not differ in their willingness to pay, and therefore they receive the same treatment from lenders. Political instability affects the equilibrium spread through its effect on the weight on future utility flows. Another related work is the paper by Chang (2007). He studies the simultaneous determination of financial default and political crises in an open economy model. Political crises accompany default in equilibrium because of an information transmission conflict between the government and the public.

As in Kovrijnykh and Szentes (2007), the benchmark model predicts delay in reaching an agreement after a default. In the model studied in this paper, consistent with the evidence they show, incentives to repay are stronger in good times. In Kovrijnykh and Szentes (2007) the time elapsed from the default period to a return to markets is also endogenous. However, their model predicts periods of exclusion potentially long. Pitchford and Wright (2007) analyze what is the optimal tradeoff between efficient borrowing ex ante and the cost of default ex post. They present a model of sovereign borrowing default coupled with an explicit model of debt restructuring process in which delay arises due to both creditor holdout and free-riding on negotiation effort.

Default does not arise in equilibrium and the incentive to default is higher during good times in papers such as Kehoe and Levine (1993), Kocherlakota (1996), Alvarez and Jermann (2000) and Kehoe and Perri (2002). This paper differs from the literature on endogenous incomplete markets because in the model presented here default occurs in equilibrium and incentives to default are higher during low income realizations.

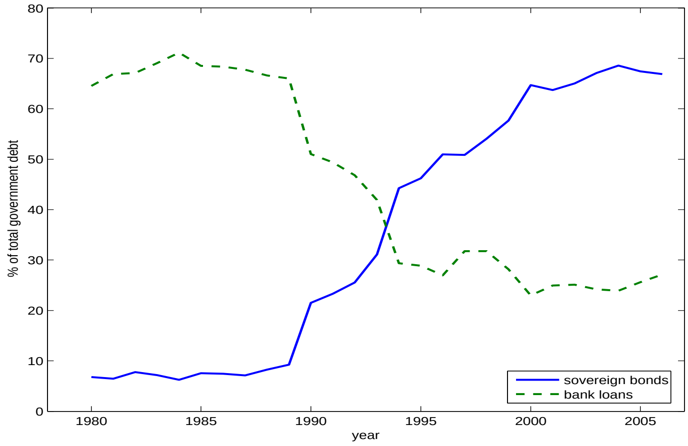
The rest of the chapter is organized as follows. In Section 1.1.2 I present a description of the sovereign debt market in emerging economies. In section 1.2, I describe the environment. In section 1.3, I describe and define the equilibrium. Section 1.4, presents the computation algorithm and the estimation procedure. Finally, in sections 1.5 through 1.8, I explain the main results.

1.1.2 The Emerging Economies Bond Market

During the past decades the market for sovereign lending has shift from syndicated bank loans towards bonds and has opened up the market to larger number of investors. Figure 1.1 shows the evolution of government debt by type (bonds vs bank loans) as a fraction of total government debt in emerging economies as a fraction of total debt. We observe a sharp decrease of the bank debt towards sovereign bonds specially since the end of the 1980's.

Unlike loan restructurings, no formal mechanisms for sovereign bond renegotiation, or workouts, have been established. Markets have addressed the issues of bond workouts on a case-by-case basis, and essentially without intervention by creditor countries or multilateral institutions (see Chuhan and Sturzenegger (2005)). Two approaches to sovereign bond workouts have been followed: voluntary and involuntary (or concerted). Voluntary exchanges between a government and lenders typically re-profile debt service, but do not lower the nominal value of debt and impose small net present value reductions. A concerted renegotiation will involve a

Figure 1.1: Sovereign Debt in Emerging Economies by Type



Source: World Bank (2007).

reduction in the net present value of the investment for investors. Table 1.2 displays the recovery rates and periods of exclusion during the most recent sovereign defaults. The sovereign ratings of these countries were always downgraded to the state of Selective Default (SD) or Default (D) until the repayment of the renegotiated debt. The recovery rates correspond to the difference between the present value of the new instruments versus the present value of the old instruments (see Sturzenegger and Zettelmeyer (2005) and Moody's (2006) for a comprehensive explanation).[6]

In response to the increased demand for the evaluation of creditworthiness, several agencies such as Moodys and Standard & Poor's have developed expertise in estimating country risk.[7] Sovereign ratings have become an important component of the sovereign debt market. Standard & Poor's (2006) reports that a sovereign

[6]While the renegotiation mechanism is different, recovery rates for rated corporate defaults are similar. Moody's (2006) documents that the average senior unsecured bond recovery rate is 58.3% in 2006.

[7]By 1970 S&P's rated only two countries, U.S. and Canada. The number of rated sovereigns rose to 12 in 1980, all rated 'AAA'. From that point on, there was a marked increase in the number of ratings and an expansion into lower rating categories.

Table 1.2: Recovery Rates and Default Episodes

Country	Date of default	Time in default (months)	Recovery rate (%)
Russia	09/98	26	45
Pakistan	10/98	14	69
Ecuador	10/99	12	73
Ukraine	01/00	2	72
Argentina	11/01	42	27
Indonesia	04/02	5	na
Paraguay	02/03	14	na
Uruguay	04/03	2	87
Grenada	12/04	10	65
Venezuela	01/05	2	na
Average		13	62

Sources: Sturzenegger and Zettelmeyer (2005); Moody's (2006); and Standard & Poor's (2007).

rating aggregates information from two main sources: economic and political risk. Both factors determine the sovereign's willingness to repay. The evidence shows that ratings directly affect the extension to government credit. Table (1.3) displays the correlation of sovereign ratings and country's spread over time observed for emerging economies[8]. Ratings have been good predictors of defaults. Countries in the lowest categories account for more than 70 percent of the total number of default episodes in a one year horizon (See Standard & Poor's and Moody's).

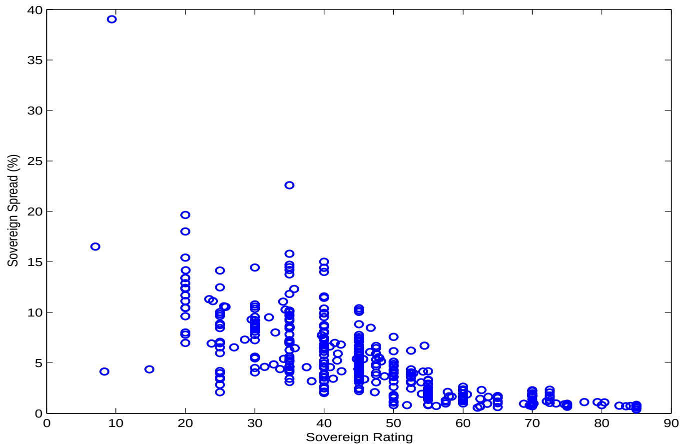
[8] A country's spread is the difference between a bond of the country and that of similar maturity in the U.S. For the table, the an index generated by JP Morgan, EMBI, is used to calculate the correlation with the ratings. Ratings are sovereign ratings reported by Standard & Poor's and transformed into numbers using a linear scale.

Table 1.3: Sovereign Ratings and Spreads

Country	Correlation Spreads and Ratings over time
Argentina	-0.86
Brazil	-0.32
Chile	-0.77
Ecuador	-0.75
Korea	-0.80
Malaysia	-0.85
Mexico	-0.82
Turkey	-0.82
Venezuela	-0.35

Sources: JP Morgan (2007); and Standard & Poor's (2007).

Figure 1.2: Sovereign Ratings and Interest Rates



Ratings affect the extension of government credit in four ways. First, a higher rating implies lower interest rates. Second, the presence of a "selective default" or "default" flag, constrains a country from market access. Third, after a default, countries emerge with lower ratings than before the episode. Fourth, countries that borrow more receive lower ratings. Higher credit ratings translate into lower country spreads (Cantor and Packer (1996) and Eichengreen and Mody (2000)). For these reasons, upgrades or downgrades in sovereign ratings have considerable impact in countries access to credit. Figure 1.2 shows the relation between rating and spreads for the group of countries in the previous table during the last decade. Each point corresponds to a year-country pair. One can observe a clear negative relation.

1.2 Environment

I study sovereign default and debt renegotiation in a dynamic small open economy with private information. The model consists of a large number of competing lenders and a sovereign that borrows from them. The country receives a stochastic stream of income and debt contracts are restricted to one-period bonds. Debt contracts are not enforceable.

1.2.1 Preferences and Endowment Process

Preferences of the representative consumer in period τ are given by

$$\mathbb{E}_\tau \sum_{t=\tau}^{\infty} \beta^t u(c_t) \tag{1.1}$$

where $u(c)$ is strictly concave and differentiable, $\beta \in (0,1)$ is the discount factor and c_t is aggregate consumption in period t. The endowment of the consumption good that the agents receive in each period is denoted by $y_t \in Y \subseteq \mathbf{R}_{++}$. I assume that y_t follows a Markov process with conditional distribution function $F(y_{t+1}|y_t)$.

International lenders are risk neutral. They can borrow or lend as much as necessary at the risk free rate r in international markets.

1.2.2 Government

The government chooses consumption to maximize the expected discounted utility of the representative consumer. As in Cole, Dow and English (1995) and Phelan

(2006), I assume that the government transitions between two states. Political uncertainty, changes in the composition of the leading coalition, or changes in the distribution of power within the government generate that government's valuation of the future varies over time. The political state of the government is private information and changes over time. According to these states we can classify the government as "aligned" (the government discounts the future at the same rate as consumers, i.e. $\delta_a = 1$) or "misaligned" (the government discounts the future at a lower rate, i.e. $\delta_m < 1$). No restrictions are imposed on the value of δ_m besides $\delta_m \in [0, 1)$. The political state evolves according to a first order markov process with the transition matrix given by:

$$\Pi = \begin{bmatrix} \pi_a & 1 - \pi_a \\ 1 - \pi_m & \pi_m \end{bmatrix}, \tag{1.2}$$

where π_i is the probability that at state i the government will stay at the same state in the following period. The transitions between government states are not observed by the lenders. For future reference, I will denote the unconditional probability of a government being an $a - type$ by

$$\pi^* = \frac{(1 - \pi_m)}{(1 - \pi_a) + (1 - \pi_m)}. \tag{1.3}$$

1.2.3 Information Structure and Strategies

The credit industry accepts deposits and makes loans to the sovereign country. Asset markets are not complete. In each period, the government borrows or saves using only one period non-contingent bonds denoted by b. Let x and x' be any variable x at period t and $t + 1$ respectively. The set of possible values of b is $B \equiv [\underline{b}, \overline{b}] \subset \mathbf{R}$, with $\underline{b} < 0 < \overline{b}$. When the government borrows from international markets b' will take negative values, $b' < 0$.

The level of debt, output and the actions of the government are observable. However, the political state is private information and lenders need to infer the sovereign's state based on its actions. I will denote by s the lender's posterior probability that the government is of the $a - type$ at the given observable state (b, y). In a markov perfect bayesian equilibrium, the actions of the players will be specified as function of s.

The government borrows at price $q(b', s', y)$ where b' is the borrowing level, s' is the market assessment of the government type, i.e. the probability that the government is of the $a - type$ and y is the current output realization. If the government borrows from foreign lenders, it receives $-q(b', s', y)b'$ units of current period goods and promises to deliver b' units of the following period good. The government is not committed to repay its debts. In equilibrium, the price function will reflect the risk of default and the recovery value of the defaulted debt. A government that defaults will remain in financial autarky until it repays an endogenously determined fraction of the defaulted debt. The variable h will denote the government status in international credit markets, $h \in \{In, Out\}$. If $h = In$, the government does not have an unresolved default and it is free to borrow or save in credit markets. On the other hand, if $h = Out$ the government still has to resolve an old default and it is not allowed to borrow or save.

In the default period, total consumption equals the endowment. If a country has an unresolved default, $h = Out$, and unpaid debt $b < 0$, the country is subject to exclusion from credit markets and the default situation entitles a proportional output loss $\lambda \in (0, 1)$. If the government decides to repay, the present value of the debt is reduced to a fraction $\phi(b, y, s; \sigma)$ of its original size. This fraction is determined at the renegotiation stage between the government and the lender and depends on the observable state in the repayment period (b, y, s) and on the probability that the lenders assign to the government they are facing in the renegotiation stage is of the

$a - type$, σ.

The government and the lenders bargain over the debt recovery rate $\phi(b, y, s; \sigma)$. At the renegotiation stage, a Nash bargaining solution is defined in this environment with incomplete information. The properties derived from this mechanism are consistent with what we observe in the data and are explained in more detail in the next section. After the agreement, the present value of the debt is reduced by a fraction $\phi(b, y, s; \sigma)$ of its original size b.

1.2.4 Time Line

The timing of events can be summarized as follows: at the beginning of each period, the government inherited a level of debt b and credit history (s, h). The government's type and the country's endowment are realized. Only the endowment is revealed to the lenders. When $h = In$ the government decides to default or not. If it decides not to default, it chooses how much to borrow or lend b' at price $q(b', s', y)$ for the following period. If it chooses to default, consumption equals the endowment and $h' = Out$. If $h = Out$ the government has an unresolved default and has to decide to renegotiate or not. If the government repays the reduced debt, $\phi(b, y, s; \sigma)b$, it regains access to the credit market in the following period ($h' = In$). In the case the government does not repay the old debt it will stay out of credit markets, $h' = Out$. Based on government actions the lenders will update their information about the sovereign's type, i.e. when $h = In$, s' is a function of the current observable government state (b, y, s), the default decision $d \in \{0, 1\}$ and the level of borrowing b'. When $h = Out$ the posterior s' is a function of (b, y, s), the repayment decision $z \in \{0, 1\}$ and the recovery rate $\phi(b, y, s; \sigma)$.

1.3 Equilibrium

In this paper, I focus on Markov Perfect Bayesian Equilibria, and strategies are restricted to be stationary functions from beliefs (or more precisely posterior probabilities) to actions. Beliefs and strategies need to be specified for all possible states, even those that never happen in equilibrium. Markov strategies are defined relative to the state variable s, the sovereign rating.

Definition 1 *A* ***Stationary Markov Strategy*** *for a government in political state i is a set of stationary functions: consumption, $c_i(b, y, s, h)$; debt accumulation, $b'_i(b, y, s)$; default, $d_i(b, y, s)$ and debt repayment $z_i(b, y, s)$.*

At observable state $(b, y, s, h = In)$, the probability that lenders assign to a government of being of the $a - type$ after observing his debt, $b' \in B$, and default decisions, $d \in \{0, 1\}$, is denoted by $\sigma^{In}(b', d, b, y, s)$, that is

$$\sigma^{In}(b', d, b, y, s) \equiv \Pr(a|b', d, b, y, s; h = In).$$

If the government is out of the credit market, i.e. at state $(b, y, s, h = Out)$, the probability that it is of type a, conditional on the repayment decision $z \in \{0, 1\}$, is denoted by $\sigma^{Out}(z, b, y, s)$:

$$\sigma^{Out}(z, b, y, s) \equiv \Pr(a|z, b, y, s; h = Out).$$

This probabilities are defined for allocations on and off-the equilibrium path. The possibility of a type change implies that the sovereign rating (the posterior) at the beginning of the following period is:

$$\Psi^{In}(b', d, b, y, s) = \pi_a \sigma^{In}(b', d, b, y, s) + (1 - \pi_b)(1 - \sigma^{In}(b', d, b, y, s)), \qquad (1.4)$$

and

$$\Psi^{Out}(z,b,y,s) = \pi_a \sigma^{Out}(z,b,y,s) + (1-\pi_b)(1-\sigma^{Out}(z,b,y,s)). \tag{1.5}$$

1.3.1 Government's Problem

The objective of the government is to maximize the expected discounted utility of the representative consumer. The government solves the following problem: If it is active in credit markets and has debt, it chooses to default or not and in the case of no-default how much to borrow or lend. If it is active but it does not have debt, it only chooses how much to borrow or save. If it is not active it chooses to renegotiate or not. The state variables for the government are its political state $i \in \{a, m\}$, the level of assets inherited from the previous period b, the endowment y, the probability the lender assigns to the government of being of the good type s and credit history h. Let the value function of a government of type i at state (b, y, s, h) be denoted by $V_i(b, y, s, h)$. $V_{-i}(b, y, s, h)$ denotes the value function of the political state different than i.

If $b < 0$ and $h = In$, the government can choose between repaying its outstanding debt or defaulting. If the government settles the debt, it chooses its next period debt level b' and consumes. If it defaults, it cannot borrow or save and consumes its endowment. Based on its actions, the sovereign rating is updated to $s' = \Psi^{In}(b', d, b, y, s)$ and h' is set to In or Out if $d = 0$ or $d = 1$ respectively. The value function is

$$V_i(b, y, s, In) = \max \Big\{ v_i^{nd}(b, y, s), v_i^{d}(b, y, s) \Big\}. \tag{1.6}$$

$v_i^{nd}(b, y, s)$ denotes the value of "no-default":

$$v_i^{nd}(b,y,s) = \max_{c\geq 0, b'\in B} \left\{ u(c) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(b',y',s',In) + (1-\pi_i)V_{-i}(b',y',s',In)\Big] \right\} \quad (1.7)$$

$$\begin{aligned} \text{s.t. } c + q(b',s',y)b' &\leq y + b, \\ s' &= \Psi^{In}(b', d=0, b, y, s). \end{aligned}$$

$v_i^d(b,y,s)$ is the value of "default":

$$v_i^d(b,y,s) = u(y) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(b,y',s',Out) + (1-\pi_i)V_{-i}(b,y',s',Out)\Big], \quad (1.8)$$

$$\text{s.t. } s' \quad = \quad \Psi^{In}(b, d=1, b, y, s).$$

If $b \geq 0$ and $h = In$, the government does not have debt, so there is no default decision to make. In this case, the value for a government $i - type$ is given by

$$V_i(b,y,s,In) = v_i^{nd}(b,y,s). \quad (1.9)$$

If $h = Out$ the country is out of the credit market and has some unpaid debt $b < 0$. While the country is in this state, it suffers a proportional output loss of size λy. Only if the government repays the old debt, reduced to $\phi(b,y,s;\sigma)b$, it will regain access to credit markets. The recovery rate $\phi(b,y,s;\sigma)$ is endogenously determined in the bargaining process and can take any value in $[0,1]$. The problem that the government solves is:

$$V_i(b,y,s,Out) = \max\Big\{v_i^r(b,y,s), v_i^{nr}(b,y,s)\Big\}. \quad (1.10)$$

$v_i^r(a, y, s)$ denotes the value of "repay":

$$v_i^r(b, y, s) = u(y+\phi(b, y, s; \sigma)b)+\beta\delta_i E_{y'|y}\Big[\pi_i V_i(0, y', s', In)+(1-\pi_i)V_{-i}(0, y', s', In)\Big] \tag{1.11}$$

$$\text{s.t.} \quad s' \quad = \quad \Psi^{h=Out}(z = 1, b, y, s).$$

$v_i^{nr}(b, y, s)$ denotes the value of "no-repay":

$$v_i^{nr}(b, y, s) = u(y(1-\lambda)) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(b, y', s', Out) + (1-\pi_i)V_{-i}(b, y', s', Out)\Big] \tag{1.12}$$

$$\text{s.t.} \quad s' \quad = \quad \Psi^{h=Out}(z = 0, b, y, s).$$

The solution to this problem provides the value function $V_i(a, y, s, h)$ for $i = a, m$ and $\forall$ (b, y, s, h) and the optimal choices, $b' = g_i(b, y, s)$, $d_i(b, y, s)$, and $z_i(b, y, s)$ of assets, default and repayment respectively. These optimal choices allow me to characterize the default set $D_i(b, s) \subseteq Y$ and the renegotiation set $Z_i(b, s) \subseteq Y$ for a government in political state i as follows:

$$D_i(b, s) = \{y \in Y : d_i(b, y, s) = 1\}, \tag{1.13}$$

and

$$Z_i(b, s) = \{y \in Y : z_i(b, y, s) = 1\}. \tag{1.14}$$

For future reference, I can also define the set of endowments for which a government in political state i with posterior s chooses b':

$$E_i(b', b, s) = \{y \in Y : d_i(b, y, s) = 0 \ \& \ g_i(b, y, s) = b'\}. \tag{1.15}$$

1.3.2 Foreign Lender's Problem

Foreign creditors can borrow or lend at the risk free-rate $r \geq 0$. The market for sovereign bonds is competitive so they take the price schedule $q(b', s', y)$ as given. Recall from equation (1.4) that after choosing b' the government is thought to be of the $a-type$ at the beginning of the following period with probability $s' = \Psi^{In}(b', d = 0, b, y, s)$. From the default set defined in (1.13) it is possible to derive the equilibrium probability of default on a loan b' when the government is at state (b, y, s):

$$p(b', s', y) = \int_{D_a(b', s')} F(dy'|y)s' + \int_{D_m(b', s')} F(dy'|y)(1 - s').$$

The default probability takes into account the possible type change at the end of the period as well as the transition to different endowments levels. With probability $s' = \Psi^{In}(b', d = 0, b, y, s)$ the government is of the $a - type$ in the following period and will default in endowments states $y' \in D_a(b', s')$. Similarly, with probability $(1 - s') = (1 - \Psi^{In}(b', d = 0, b, y, s))$ the government is of the $m - type$ in the following period and will default in endowments states $y' \in D_m(b', s')$.

The profit on a loan of size b' made to a government with assets b, endowment y and current rating s, denoted by $\Omega(b', s', y; p, \rho)$, equals the expected present discounted value of inflows less the current value of outflows. It depends on the price $q(b', s', y)$, on the probability of the government defaulting on it, $p(b', s', y)$, and on the expected present value of loan repayments after a default, $\rho(b, s, y)$. In equilibrium, the function $\rho(b, s, y)$ is derived from the decision rules of the government and the posterior function. Consider a situation where a government with prior s is in financial autarky. At the beginning of a period, before any action is taken, lenders expects to receive:

$$[sz_a(b, y, s) + (1 - s)z_m(b, y, s)]\phi(b, y, s; \sigma)(-b),$$

that is, with probability s, lenders are facing an $a - type$ government and they will receive $\phi(b, y, s; \sigma)(-b)$ if the government chooses to repay, i.e. if $z_a(b, y, s) = 1$. Similarly, with probability $(1 - s)$ they are facing a $m - type$ government and they will receive $\phi(b, y, s; \sigma)(-b)$ if the government chooses to repay, i.e. if $z_m(b, y, s) = 1$. Recursively, the expected recovery value on a defaulted loan is

$$\begin{aligned}\rho(b, s, y) &= [sz_a(b, y, s) + (1 - s)z_m(b, y, s)]\phi(b, y, s; \sigma)(-b) \\ &+ \frac{1}{(1 + r)}[s(1 - z_a(b, y, s)) + (1 - s)(1 - z_m(b, y, s))]E_{y'|y}[\rho(b, s', y')],\end{aligned} \tag{1.16}$$

where the posterior s' corresponds to a government that chooses not to repay, i.e. $s' = \Psi^{Out}(z = 0, b, y, s)$. Hence, the profit on a loan of size b' made to a government with assets b, endowment y and current rating s, $\Omega(b', s', y; p, \rho)$ is

$$\begin{aligned}\Omega(b', s', y; p, \rho) = -q(b', s', y)(-b') + \frac{(1 - p(b', s', y))(-b')}{(1 + r)} + \\ \frac{p(b', s', y)}{(1 + r)^2}E\rho(b', s'', y''),\end{aligned} \tag{1.17}$$

where $s'' = \Psi^{In}(b', d' = 1, b', y', s')$ is the posterior for the government after a default.

In most of the recent models of sovereign debt, it is assumed that countries obtain full discharge of debt after a default (see for example Arellano (2007) and Aguiar and Gopinath (2006)). This implies that, in these papers, the last term in expression (1.17) always equals zero. The exception is the paper by Yue (2007) that considers endogenous debt renegotiation in an environment with full information. However, in that paper, there is no delay in reaching an agreement and debt reduction is immediate because countries are forced to renegotiate in the default period.

Perfect competition implies that $\Omega(b', s', y; p, \rho) = 0$ and the equilibrium price function is:

$$q(b', s', y; p, \rho) = \begin{cases} (1+r)^{-1} & \text{if } b' \geq 0, \\ \frac{(1-p(b',s',y))}{(1+r)} + \frac{p(b',s',y)E_{y''|y}\rho(b',s'',y'')}{(-b')(1+r)^2} & \text{if } b' < 0. \end{cases}$$

This expression shows why the probability of default plays an important role in determining the equilibrium price. As expected, a higher probability implies a higher interest rate (a lower price). However, the value of the expected recovery value $\rho(b', s'', y'')$ sets a limit to the lowest value of the contract. At a given default probability, higher recovery rates will imply lower interest rates. Since $p(b', s', y) \in [0, 1]$ and $0 \leq E_{y'|y}[\rho(b', s'', y'')/(-b')] \leq 1$ the set of feasible prices is $Q = [0, (1+r)^{-1}]$.

1.3.3 Debt Renegotiation: A Nash Mechanism

In an environment with full information, a "reasonable" outcome is the Nash bargaining solution because it is pairwise Pareto-efficient. In contrast, for meetings with incomplete information very few guidelines exist for the choice of the mechanism. Following Berentsen and Rocheteau (2004), I choose a particular mechanism, which maximizes the product of the expected surplus of the government and the lenders, given their beliefs, that coincides with the Nash bargaining solution when there is complete information. This mechanism has many implications consistent with what we observe in the data. These are explained in more detail in the section with the estimation results.

Upon the bargaining agreement, the present value of defaulted debt is reduced to a fraction φ of the unpaid debt b. The value of an agreement of size φ to a government in political state i is

$$v_i^r(b, y, s; \varphi) \quad = \quad u(y + \varphi b) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(0, y', s', In) + (1 - \pi_i)V_{-i}(0, y', s', In)\Big],$$

that is the expected life time utility of repayment at state (b, y, s) when the debt recovery is φ. The value of the posterior is $s' = \pi_a \sigma + (1 - \pi_m)(1 - \sigma)$ where σ denotes the prior probability that lenders' assign to the government they are facing at the bargaining stage of being of the $a - type$.

In order to keep the model tractable, I assume that the threat point of the government in political state i is permanent autarky with output cost λy. Recursively, the autarky value $v_i^{aut}(y)$ for government of type i at income level y is

$$v_i^{aut}(y) = u(y(1-\lambda)) + \beta \delta_i E_{y'|y}\Big[\pi_i v_i^{aut}(y') + (1-\pi_i) v_{-i}^{aut}(y')\Big].$$

Thus, for a government of the $i - type$, the surplus of an agreement is

$$\Delta_i^B(\varphi; b, y, s) = v_i^r(b, y, s; \varphi) - v_i^{aut}(y). \tag{1.18}$$

The surplus of the agreement for the lenders is the present value of the recovered debt:

$$\Delta^L(\varphi; b, y, s) = -\varphi b. \tag{1.19}$$

If lenders have all the bargaining power, then they could extract all the government's surplus and viceversa. To analyze a general case, I assume that the government has bargaining power θ and the lender has bargaining power $(1 - \theta)$. The bargaining power is independent of σ and thus is not affected by the timing of the repayment decision. The optimal debt recovery rate $\phi(b, y, s; \sigma)$ satisfies the following program:

$$\phi(b, y, s; \sigma) \equiv \arg\max_{\varphi \in [0,1]} [\sigma \Delta_a^B(\varphi; b, y, s) + (1-\sigma)\Delta_b^B(\varphi; b, y, s)]^{\theta} [\Delta^L(\varphi; b, y, s)]^{1-\theta} \tag{1.20}$$

$$\begin{aligned} \text{s.t. } \Delta_a^B(\varphi; b, y, s) &\geq 0, \\ \Delta_m^B(\varphi; b, y, s) &\geq 0, \\ \Delta^L(\varphi; b, y, s) &\geq 0. \end{aligned}$$

Under this mechanism, the resulting offer maximizes the product of the expected surpluses of the players given their prior beliefs subject to the corresponding participation constraints of the government of type a, the government of type m and the lenders. The solution to this mechanism is a single offer. Hence, by construction, the value of φ that maximizes (1.20) is also incentive compatible.

Note that, in equilibrium, agreements are reached in one period. However, the period of financial exclusion is still endogenous in this model. At a given recovery schedule $\phi(b, y, s; \sigma)$, the government will optimally choose when to start the renegotiation process. The expected duration of financial exclusion increases with the equilibrium repayment fraction.

1.3.4 Definition of Equilibrium

A Markov Perfect Bayesian Equilibrium (MPBE) requires that, at every possible state (including those that only occur in off-the-equilibrium-path), agents' beliefs over the types and strategies of the other agents must be specified. Given these beliefs, each agent must choose actions that are the best responses to the strategies of the other agents. The government and the lenders only use stationary Markov strategies.

Definition 2 *A MPBE is a set of functions $V_i^*, \sigma^*, q^*, p^*, \rho^*, \phi^*$ and sets D_i^*, E_i^* and Z_i^* such that:*

1. *Given posterior functions, prices and the recovery rate, the value functions V_i^* and the sets D_i^*, E_i^* and Z_i^* for $i = a, m$, are consistent with the government's optimization problem.*

2. *Given posterior functions, prices and value functions* V_i^* *for* $i = a, m$, *the recovery rate* $\phi(b, y, s; \sigma^*)$ *is derived from the bargaining mechanism.*

3. *The equilibrium default probability* $p^*(b', s', y)$ *and expected recovery* $\rho^*(b', s', y)$ *are consistent with the government decision rules and the bargaining solution.*

4. *The equilibrium price is such that foreign creditors earn zero profits in expected value, that is at* $q^*(b', s', y)$

$$\Omega^*(b', s', y; p^*, \rho^*) = 0, \quad \textit{for all } (b', s', y),$$

5. *The function* σ^* *must be consistent with Bayes' rule (whenever possible) and posteriors are defined as in equations (1.4) and (1.5).*

The definition of equilibrium is standard. Condition 5 defines the function σ^* and it deserves a detailed explanation. This function must be consistent with Bayes' rule *whenever* applicable. When $h = In$, the probability that a government is type a conditional on their asset market behavior was denoted by $\sigma^{h=In}(b', d, b, y, s)$, and can be written as

$$\begin{aligned}\sigma^{In}(b', d, b, y, s) &= \frac{\Pr(b', d, b, y, s|a)\Pr(a)}{\Pr(b', d, b, y, s)} \\ &= \frac{\Pr(b', d|a, b, y, s)\Pr(a|b, y, s)}{\sum_i \Pr(b', d|i, b, y, s)\Pr(i|b, y, s)},\end{aligned} \tag{1.21}$$

provided $\sum_i \Pr(b', d|i, b, y, s)\Pr(i|b, y, s) > 0$. When the conditioning set is empty, the definition of equilibrium does not impose any restriction. In the computation and estimation of the model, I set the off-the-equilibrium path posteriors σ as follows: if the government defaults is believed to be of the misaligned type; if the government does not default and increases or maintains the level of debt is also believed to be of the misaligned type; if the government does not default and decreases the level of debt is believed to be of the aligned type; if repays it is believed to be of the aligned type.[9]

Prior to the government's type realization, the posterior probability that a government with observable state $(b, y, s, h = In)$ who also defaults on his loan, is of type a is given by

$$\sigma^{In}(b, d = 1, b, y, s) = \begin{cases} 1 & \text{if } y \in D_a(b, s) \text{ and } y \notin D_m(b, s), \\ 0 & \text{if } y \notin D_a(b, s) \text{ and } y \in D_m(b, s), \\ s & \text{if } y \in D_a(b, s) \text{ and } y \in D_m(b, s). \end{cases}$$

The posterior function $\sigma^{h=In}(b, d = 1, b, y, s)$ is defined *off-the-equilibrium* path when $y \notin D_a(b, s)$ and $y \notin D_m(b, s)$ because Bayes' rule is not applicable. One of the main differences between the environment in this paper and that in Chatterjee et. al. (2007) is that income is observable. This translates into a simpler posterior function, i.e. at a given state (b, y, s), $\sigma^h(\cdot)$ can take only three values: 1, 0 or s.[10]

The probability that a government with observable state $(b, y, s, h = In)$ who does not default on his loan and chooses b' is of type a is given by

$$\sigma^{In}(b', d = 0, b, y, s) = \begin{cases} 1 & \text{if } y \in E_a(b', b, s) \text{ and } y \notin E_m(b', b, s), \\ 0 & \text{if } y \notin E_a(b', b, s) \text{ and } y \in E_m(b', b, s), \\ s & \text{if } y \in E_a(b', b, s) \text{ and } y \in E_m(b', b, s) \end{cases}$$

[9]The estimation results that I present later are robust to different assumptions about the off-the-equilibrium path beliefs.

[10]Multiple computations show that existence of an equilibrium is still an issue but the space of parameters for which an equilibrium does not exists is considerably reduced.

Similarly, when $h = Out$, after observing the decision to repay or not, the probability that the government is of the $a - type$ is:

$$\begin{aligned}\sigma^{Out}(z,b,y,s) &= \frac{\Pr(z,b,y,s|a)\Pr(a)}{\Pr(z,b,y,s)} \\ &= \frac{\Pr(z|a,b,y,s)\Pr(a|b,y,s)}{\sum_i \Pr(z|i,b,y,s)\Pr(i|b,y,s)},\end{aligned}$$

provided $\sum_i \Pr(z|i,b,y,s)\Pr(i|b,y,s) > 0$. Hence, at $h = Out$, when the government repays the probability that it is of the $a - type$ is:

$$\sigma^{Out}(z=1,b,y,s) = \begin{cases} 1 & \text{if } y \in Z_a^*(b,s) \text{ and } y \notin Z_m^*(b,s), \\ 0 & \text{if } y \notin Z_a^*(b,s) \text{ and } y \in Z_m^*(b,s), \\ s & \text{if } y \in Z_a^*(b,s) \text{ and } y \in Z_m^*(b,s). \end{cases}$$

and when it does not repay

$$\sigma^{Out}(z=0,b,y,s) = \begin{cases} 1 & \text{if } y \notin Z_a^*(b,s) \text{ and } y \in Z_m^*(b,s), \\ 0 & \text{if } y \in Z_a^*(b,s) \text{ and } y \notin Z_m^*(b,s), \\ s & \text{if } y \notin Z_a^*(b,s) \text{ and } y \notin Z_m^*(b,s). \end{cases}$$

Once $\sigma^{In}(b', d, b, y, s)$ and $\sigma^{Out}(\phi, z, b, y, s)$ have been defined for each possible history and state, the new posterior, s', can be obtained from equations (1.4) and (1.5).

1.4 Computation and Estimation

The model is solved numerically. In this section, I describe the computation and estimation procedures. No restrictions are imposed on parameter values or equilibrium behavior. Off the equilibrium beliefs were defined in the previous section.

1.4.1 Model Specification

One period is a quarter. The utility function is assumed to be CRRA:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where γ is the coefficient of relative risk aversion. The endowment is modeled as a first order autoregressive process:

$$\ln(y_t) = \rho_y \ln(y_{t-1}) + \eta_t, \text{ with } \eta \sim N(0, \sigma_\eta).$$

The full set of 10 parameters in the benchmark model is:

$$\{r, \beta, \gamma, \delta_m, \pi_a, \pi_m, \lambda, \rho_y, \sigma_\eta, \theta\},$$

where r is the risk free rate; β is the discount factor of the representative consumer; γ is the coefficient of relative risk aversion; δ_m is parameter affecting the discount factor of the $m-type$ government; π_a and π_m are the transition probabilities; λ is the output cost during financial autarky; ρ_y and σ_η control the endowment process; and θ is the bargaining power of the government (probability that it makes the offer at the bargaining stage).

1.4.2 Parameter Values and Estimation

I classify the parameters into two groups. The first group consists of 6 parameters, $\{r, \beta, \gamma, \rho_y, \ \sigma_\eta, \lambda\}$, each of which can be pinned down independently of all other parameters by one target. The risk free rate r is set to 1.5% to match the average quarterly real return on 5 year U.S. T-bills in the last 35 years. The parameter γ is set equal to 2, a common value in the open economy literature. The discount factor $\beta = (1+r)^{-1}$. The endowment process is estimated using data from the

Ministry of Finance of Argentina from 1980Q1 to 2006Q4 which are log seasonally adjusted real GDP. The data is detrended using the HP-filter with parameter 1600. The estimated values are $\rho_y = 0.86$ and $\sigma_\eta = 0.024$. The estimated values are used to derive a finite First Order Markov process by the Tauchen-Hussey method. The number of grid points for the endowment is set equal to 21. The additional loss of output in autarky, λ is set at 2% consistent with the finding of Sturzenegger (2002) that estimates the percentage of output contraction after default using a panel of 100 countries.[11] This value of output loss during period of exclusion is also used in the papers of Aguiar and Gopinath (2006) and Yue (2007).

A second group, $\{\delta_m, \pi_a, \pi_m, \theta\}$ is jointly estimated to match target statistics of the Argentinean economy and recent default episodes. This is done by the Simulated Method of Moments. This procedure consists of minimizing the distance between data moments and moments extracted from the simulated model. That is, the parameters are chosen to minimize

$$L(\Theta) = [M^d - M^s(\Theta)]W^*[M^d - M^s(\Theta)]'$$

with respect to parameters Θ, where M^d are the moments from the data, $M^s(\Theta)$ are the moments from the simulated model at parameters Θ. W^* is positive definite and optimally derived weighting matrix. Given the potential for discontinuities in the model and the discretization of the state space, I used a simulated annealing algorithm to perform the optimization. Standard errors are computed from the derivative of the objective function with respect to the structural parameters evaluated at the point estimates. [12]

The moments used in the estimation are the default frequency, the standard

[11] In a previous version of the paper, I estimated this parameter and the point estimate was close to 2%.

[12] See Gourieroux and Monfort (1996) and Ingram and Lee (1991).

deviation of the current account to output ratio, the average period in the state of default and the ratio of the standard deviation of consumption to the standard deviation of output. Reinhart, Rogoff and Savastano (2003) report four episodes of sovereign defaults in Argentina's external debt from 1824 to 1999. In 2001, Argentina defaulted a fifth time on its external debts, making its average default frequency 2.78% or 0.68% quarterly. This value is similar to the average for emerging economies reported in Table 1.1. The current account to output ratio is computed for the period 1980q1 to 2006q4. The standard deviation is equal to 1.38%. The average period in the state of default is computed using data from Standard and Poor's (2007). This value is found to be equal to 4.33 quarters and it is also consistent with the estimates of Gelos et al. (2002), who use a large sample of emerging economies to find that during default episodes in the 1990's, economies were excluded from the credit markets for about a year. Many have documented that consumption is more volatile than output in emerging economies (see for example Neuymeyer and Perri (2005)). The relative volatility of consumption $\sigma(c)/\sigma(y)$ in Argentina for the period 1980Q1-2006Q4 is equal to 1.12.

These moments were chosen mainly for two reasons. First, it is important that the moments are informative. For instance if one of the moments is independent of the parameterization of the model, we are left to estimate an under-identified model. The size of the standard errors show that the model is identified. Second, and in order to make a fair comparison of the results in this paper to those in previous papers in sovereign debt literature the moments chosen are similar to the moments used in the calibration of Arellano (2007), Aguiar and Gopinath (2006) and Yue (2007).

1.4.3 Computation

Computation of the equilibrium requires three steps: an inner loop, where the decision problem of the government given parameter values, prices, the recovery schedule and posteriors is solved; a middle loop, where prices, the recovery schedule and posteriors are obtained; and an outside loop or estimation loop where parameter values that yield equilibrium allocations with the desired (target) properties are determined. To solve the model I use the discrete state space method. Posterior functions need to be defined for every element of the debt state space. The computational task is extremely burdensome, each equilibrium requires computing thousands of equilibrium loan prices and posteriors. Moreover, the existence of an equilibrium is not always guaranteed and this makes the estimation even more difficult.

To resemble what we observe in international credit markets, sovereign ratings are restricted to take a finite number of values, i.e. $s \in S \equiv \{s_1, s_2, \ldots, s_N\}$. Recall that the future score was specified in equations (1.4) and (1.5) and given by functions $s' = \Psi^{In}(b', d, b, y, s)$ and $s' = \Psi^{Out}(z, b, y, s)$. The technological assumption I make here is that the beginning of next period sovereign rating is

$$\hat{\Psi}^{In}(b', d, b, y, s) = \arg\min_{s_i \in S} |\Psi^{In}(b', d, b, y, s) - s_i|, \text{ when } h = In,$$

and

$$\hat{\Psi}^{Out}(z, b, y, s) = \arg\min_{s_i \in S} |\Psi^{Out}(z, b, y, s) - s_i|, \text{ when } h = Out.$$

In words, this is just saying that future scores are assigned to the grid point that is closest in absolute value. Given this technological assumption, in the computation of the model, $\hat{\Psi}^h$ is substituted everywhere I have Ψ^h. The value of N is set to 20 as in the available scales for sovereign ratings by Standard & Poor's and Moody's. Note that $s_1 = (1 - \pi_m)$ and $s_N = \pi_a$. Moreover, the Markov process for political states implies that the average value of s in the long-run is equal to π^*.

1.4.4 Estimated Parameters and Model Moments

Table 1.4 displays the parameters values that can be pinned down independently of all other parameters by one target. The estimated parameters as well as their standard errors and the model moments are reported in Table 1.5. The optimal weighting matrix in the SMM procedure is derived from the variance-covariance matrix of the moments.

Table 1.4: Model Parameters: Independent Targets

Parameter		Value	Moment
Risk free interest rate	r	0.015	Q. real return US T-bill
Discount Factor	β	0.9852	$(1+r)^{-1}$
Risk aversion	γ	2	
Autocorrelation Endowment	ρ	0.86	Argentina Output 1980-2006
Std. Dev. Error	σ_η	0.024	Argentina Output 1980-2006
Output Loss	λ	0.02	Observed output loss

The discount factor of the misaligned government is found to be only 18% lower than that of the aligned government. The degree of impatience reflects some political instability in Argentina during this period. The values of π_a and π_m imply that in the stationary equilibrium a government will be of the aligned type approximately 80% of the time. Finally, the bargaining power, θ, is estimated to be 0.57 which shows that Argentina has a more favorable position in debt renegotiation than international investors.

Table 1.5: Estimated Parameters and Moments

Parameter		Value	Std. Error
Discount Factor $m - type$	δ_m	0.818	0.012
Probability $a - type$	π_a	0.941	0.009
Probability $m - type$	π_m	0.759	0.032
Bargaining Power	θ	0.574	$8.23e - 04$

Moment	Data (%)	Model (%)
Default Probability	0.68	0.64
Std. Dev. Current Account/Output	1.38	1.45
Average exclusion period (quarters)	4.33	4.09
Relative volatility of consumption $\sigma(c)/\sigma(y)$	1.12	1.08

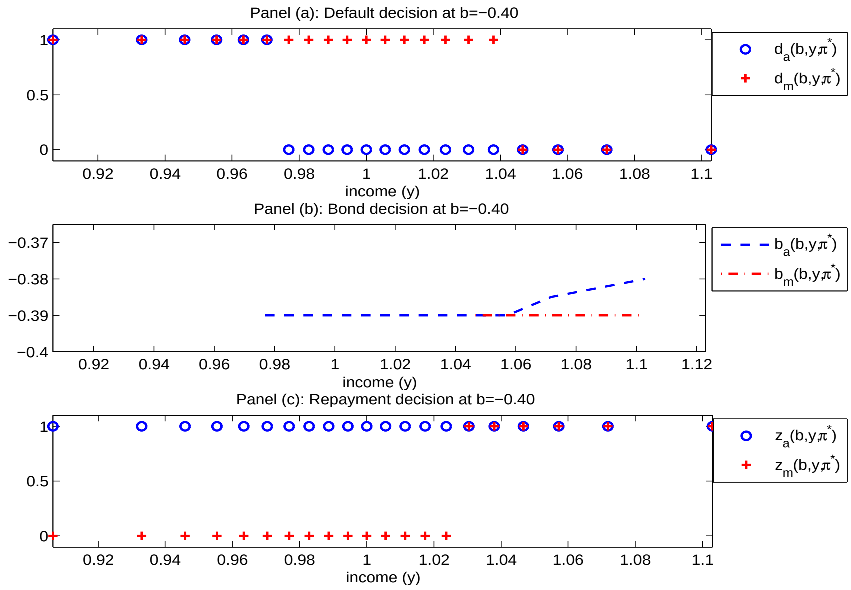
The model is able to generate the default frequency observed in the data, as well as the standard deviation of the current account to output ratio. In equilibrium, the length of the period of exclusion and the relative volatility of consumption to output are consistent with what we observe in emerging economies. In the following section, I discuss what these parameters imply for the relation between the type probabilities and the government behavior. In particular, I show that we can identify this probabilities with the sovereign rates that agencies specialized in sovereign debt construct. After that, I show what are the implications of combining government reputation and endogenous recovery rates and periods of exclusion for the debt to output ratio.

1.5 Type Probabilities as Sovereign Ratings

Sovereign ratings are a key determinant of the interest rate a country faces. In the introduction, I described the relevance of sovereign ratings in international markets and in particular how they affect the loan prices for emerging economies. In summary, ratings affect the extension of government credit in four ways. First, a higher rating implies lower interest rates. Second, the presence of a "selective default" or "default" flag, constrains a country from market access. Third, after resolving a default, ratings are upgraded. Fourth, countries that borrow more receive lower ratings. In this section, I show that the type probabilities generated by the model are consistent with the sovereign ratings observed in the data because: (*i*) the type probability s drops after a default; (*ii*) The government increases its reputation after renegotiating and repaying and old debt; (*iii*) if the government takes on more debt s decreases. Hence, from the perspective of lenders, the type probability s (the probability that the government is of the $a-type$) can be identified with a sovereign rating.

The interaction between default, borrowing and repayment is crucial to determine the relation between the type probabilities and the sovereign ratings. For example, when the government decides to default, how much debt the government takes and when it decides to borrow affect its reputation. In Figure 1.3, I plot the default, borrowing and repayment decision rules for both government types at average debt, as a function of income and evaluated at average s, that is $s = \pi^*$.

Figure 1.3: Government Decision Rules



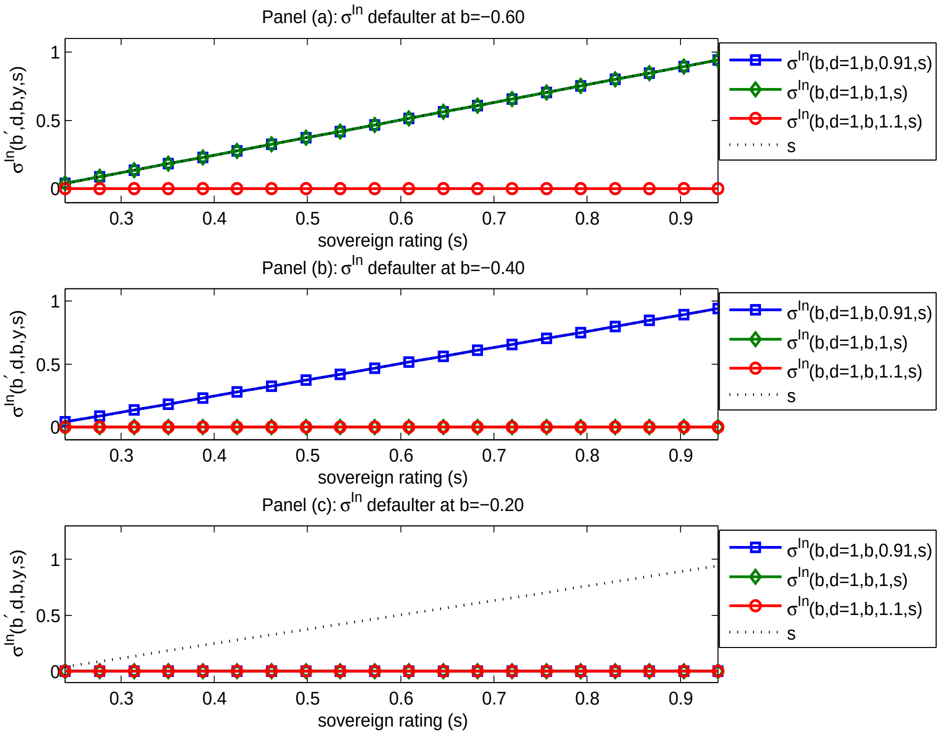
Panel (a) of Figure 1.3 shows that incentives to default are stronger in "bad times" and that the default probability is higher for a misaligned government. At low levels of income both government types decide to default when $b = -0.40$. The $a-type$ government defaults for $y < 0.97$. The $m-type$ decision rule shows a similar dependence on income levels but this government type default in more states. In particular, for $y < 1.04$ the misaligned government chooses to default. For income $y \in [0.977, 1.038]$ the $a-type$ government is able to perfectly signal its type by not defaulting. For higher levels of income the debt decision is crucial in determining the government reputation.

Panel (b) of Figure 1.3 shows the borrowing decision rule for the $a-type$ and the $m-type$ government. Recall that the government is able to borrow or save only in periods that it chooses not to default. We observe that $b_a(b, y, s) \leq b_m(b, y, s)$ for all (y) at $s = \pi^*$ and this will have also important implications for sovereign ratings. In particular, increasing the level of indebtedness have negative implications on sovereign ratings (see Figure 1.5 below). For $y \in [1.047, 1.07]$ the $m-type$ government mimics the behavior of the $a-type$ government to maintain its reputation. At $y = 1.1$ the $a-type$ government signals its type by borrowing a lower amount than the $m - type$ government. The $m - type$ government prefers to face a higher interest rate and a lower reputation than follow the $a = type$ government at this income level.

Panel (c) of Figure 1.3 displays the repayment decision rules for the $a - type$ and the $m-type$ government over income. At this debt level, the $a-type$ government chooses to repay for every level of income. By repaying when output is low the $a - type$ government is able to perfectly signal its type. The $m - type$ government repays only for $y > 1.04$. The repayment decision dependents on the level of income and, as it will be also evident in Figure 1.6 below, incentives to repay are stronger in good times. This is consistent with the data presented in Kovrijnykh and Szentes (2007) on the timing of debt repayment.

What are the main implications of the government behavior on its reputation and the posterior functions (rating functions)? I will start by describing the function $\sigma^{In}(b', d, b, y, s)$ in the case of a default, i.e. when $d = 1$. It is important to start with this function because what a government reveals by its default decision will be one of the key determinants of the interest rate it faces. Figure 1.4 displays the posterior function $\sigma^{In}(b, d = 1, b, y, s)$ as a function of the sovereign rating for different bond levels and evaluated at different income levels. From this figure we note that $\sigma^{In}(b, d = 1, b, y, s) \leq s$ for all (b, y, s). Thus, a government contemplating to default recognizes that this will lower its rating and presumably raise its future interest rates.

Figure 1.4: A Default Lowers the Sovereign Rating



Panels (a), (b) and (c) of Figure 1.4 show the function σ^{In} after a default for high, average and low debt levels respectively and evaluated at different income realizations. In Panel (a) of Figure 1.4, we observe that for debt levels $b = -0.60$, a debt level 50% higher than average debt, because both type of governments choose to default when income is equal to 0.91 or 1 (lower or equal to average income), the posterior probability that the government is of the $a - type$ equals the prior s. When income is higher than its average, $y = 1.1$, only the misaligned government defaults and the rating drops to its lowest value after breaking the debt contract. In particular, $\sigma^{In}(b, d = 1, b, 1.1, s) = 0$ for all s. The decision of no-default allows the aligned government to perfectly signal its type and increase its reputation.

Panel (b) of Figure 1.4 shows the posterior probability σ^{In} for a government that defaults at average debt $b = -0.40$. When income is at its lowest possible value

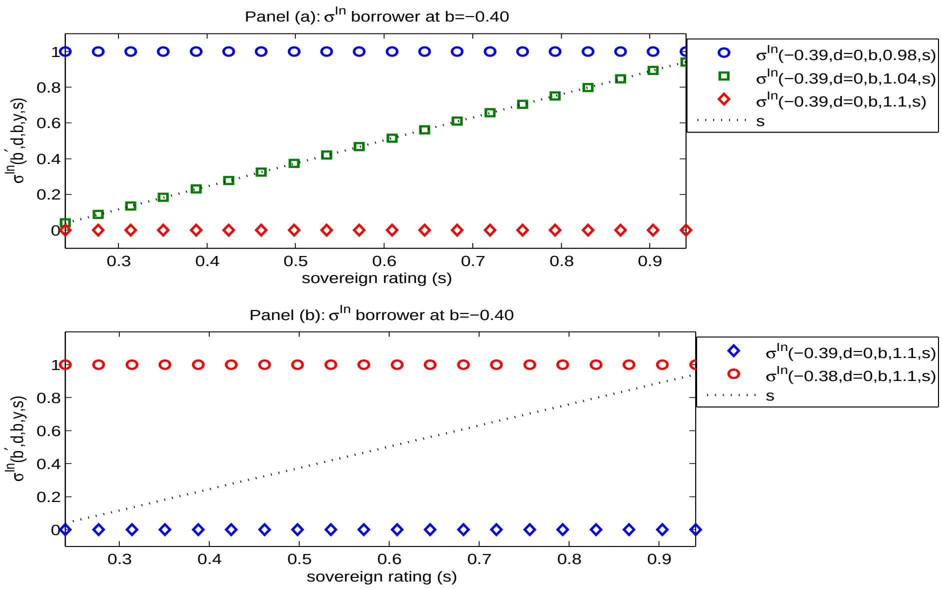
$y = 0.91$ both government types choose to default and $\sigma^{In}(b, d = 1, b, y, s) = s$. As income rises, reputation provides the incentives for the aligned type government to choose not to default. The $a - type$ government chooses not to default for any s and the $m - type$ government chooses to default for all s. In this case $\sigma^{In}(b, d = 1, b, y, s) = 0$ and the government that defaults is believed to be of the $m - type$. When $y = 1.1$, none of the government types choose to default, so posteriors are defined off-the-equilibrium path and set equal to 0, i.e. $\sigma^{In}(b, d = 1, b, y, s) = 0$. Finally, Panel (c) of Figure 1.4 shows the posterior probability σ^{In} at $b = -0.20$ (50% of average debt). When income is low, $y = 0.91$, only the $m - type$ government defaults and $\sigma^{In}(b, d = 1, b, y, s) = 0$. For higher income levels none of the government types choose to default and $\sigma^{In}(b, d = 1, b, y, s) = 0$ as defined by the off-the-equilibrium assumption on posteriors.

Evidence on emerging economies shows that the borrowing decision also affects the value of the future rating. In particular, taking on more debt negatively impacts the sovereign rating. Figure 1.5 shows that the model is also consistent with this fact. Panel (a) shows how the posterior function responds to the same level of borrowing but at different income levels. When income is $y = 0.98$ (below average) only the $m - type$ government chooses $b' = -0.39$ and $\sigma^{In}(-0.39, d = 0, b = -0.4, y = 1, s) = 1$ for all s. When income equals 1.04, both government types choose the same level of borrowing and $\sigma^{In}(-0.39, d = 0, b = -0.40, y = 1, s) = s$ for all s. When income is $y = 1.1$ (10% above average) both government types choose the same level of borrowing. In this case, only the $m - type$ government chooses $b' = -0.39$ and $\sigma^{In}(-0.39, d = 0, b = -0.4, y = 1, s) = 0$ for all s.

In Panel (b) of Figure 1.5 we observe how the posterior is affected by different borrowing levels. When $b' = -0.39$, the posterior takes value $\sigma^{In}(-0.39, d = 0, b = -0.40, y = 1, s) = 0$ because only the $m - type$ chooses this action. This government obtains a higher consumption level today at the cost of loosing its reputation. If

$b' = -0.38$, $\sigma^{In}(-0.34, d = 0, b = -0.35, y = 1, s) = 1$ and the $a - type$ government perfectly signals its type. In this case, a lower amount of debt allows the government to build a reputation. In the data, I observe that ratings are negatively correlated with the level of borrowing.

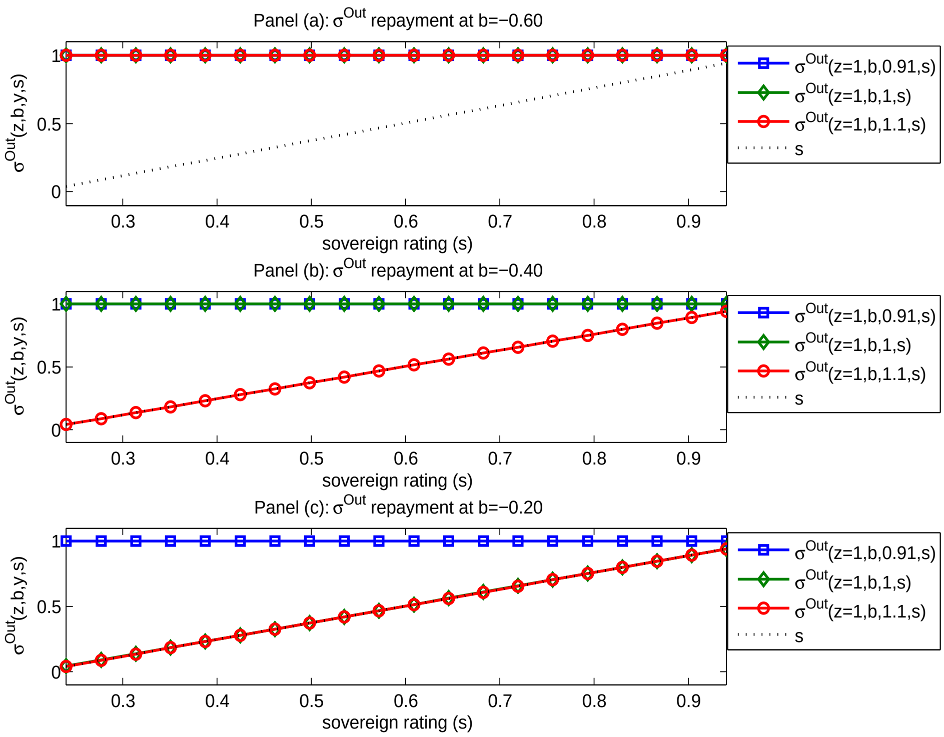
Figure 1.5: Increasing Indebtedness Lowers the Sovereign Rating



Data on sovereign ratings also show that countries increase their ratings after repayment. Also, if a government chooses not to repay, the rating is kept as its lower level. From Figure 1.6 we can infer that this is also an equilibrium outcome. In particular, $Z_m(b, s) \subseteq Z_a(b, s)$, i.e. the $a-type$ government is prone to repay more often than the $m - type$. Figure 1.6 displays the posterior function $\sigma^{Out}(z = 1, b, y, s)$, for a government that chooses to renegotiate and repay a previously defaulted debt. As it is clear from Panels (a), (b) and (c) $\sigma^{Out}(z = 1, b, y, s) \geq s$ for all (b, y, s) and in most cases, an align government will signal its type by resolving an old default.

Panel (a) of Figure 1.6 shows that when the debt level is 50% higher than average debt, $b = -0.60$, only the aligned type government chooses to repay. In this case the repayment decision signals the government type perfectly and $\sigma^{Out}(z = 1, -0.60, y, s) = 1$ for all s. From Panel (b) of Figure 1.6 we observe that, when debt is equal to average debt, $b = -0.40$, the government behaves similarly and the posterior $\sigma^{Out}(z = 1, -0.40, y, s) = 1$. Panel (c) shows that when $b = -0.20$ (50% of average debt) only the aligned government renegotiates and repays when $y = 0.91$ for all s implying that $\sigma^{Out}(z = 1, -0.20, y = 0.91, s) = 1$. For other combinations of income and ratings, both government types choose to renegotiate and repay and hence $\sigma^{Out}(z = 1, -0.20, y, s) = s$. In this case, by choosing to repay, the $m - type$ government is able to maintain its reputation.

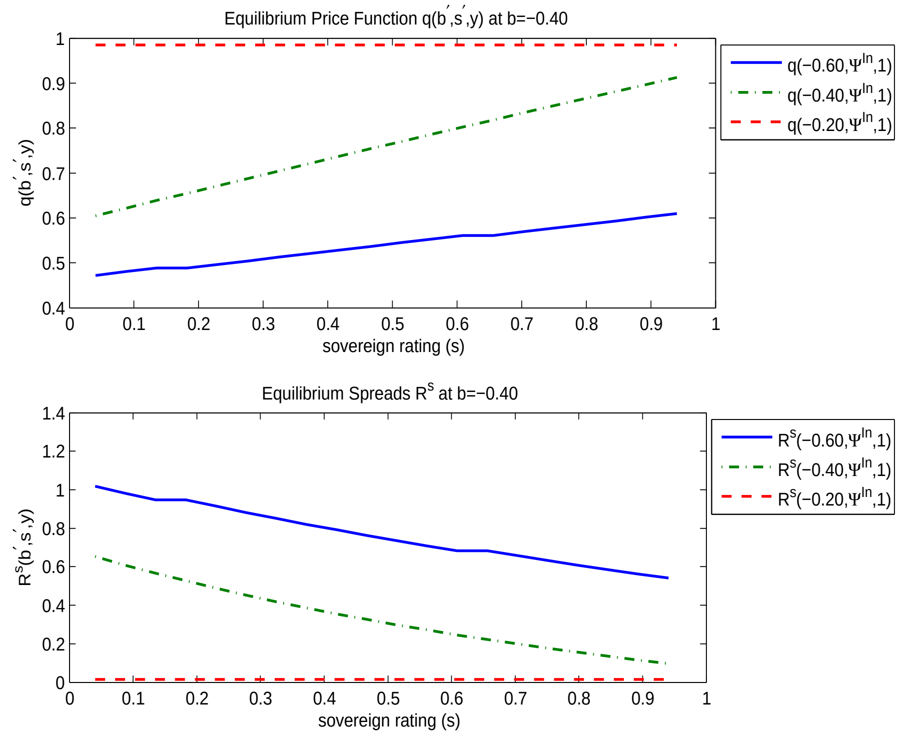
Figure 1.6: The Sovereign Rating Increases with Repayment



Next, I plot the equilibrium bond price as a function of current rating. Recall that in equilibrium the current debt level and sovereign rating affect loan prices through the posterior function, i.e. $q(b', s' = \Psi^{In}(b', d = 0, b, y, s), y)$. Panel (a) of

Figure 1.7 shows the bond price as a function of the sovereign rating s for different borrowing levels, b', evaluated at mean income and when current debt equals mean debt ($b = -0.40$). Panel (b) of Figure 1.7 shows how different bond prices over sovereign ratings translate into sovereign spreads $R^s \equiv 1/q - (1+r)$, defined as the difference between the country interest rate and the risk free rate.

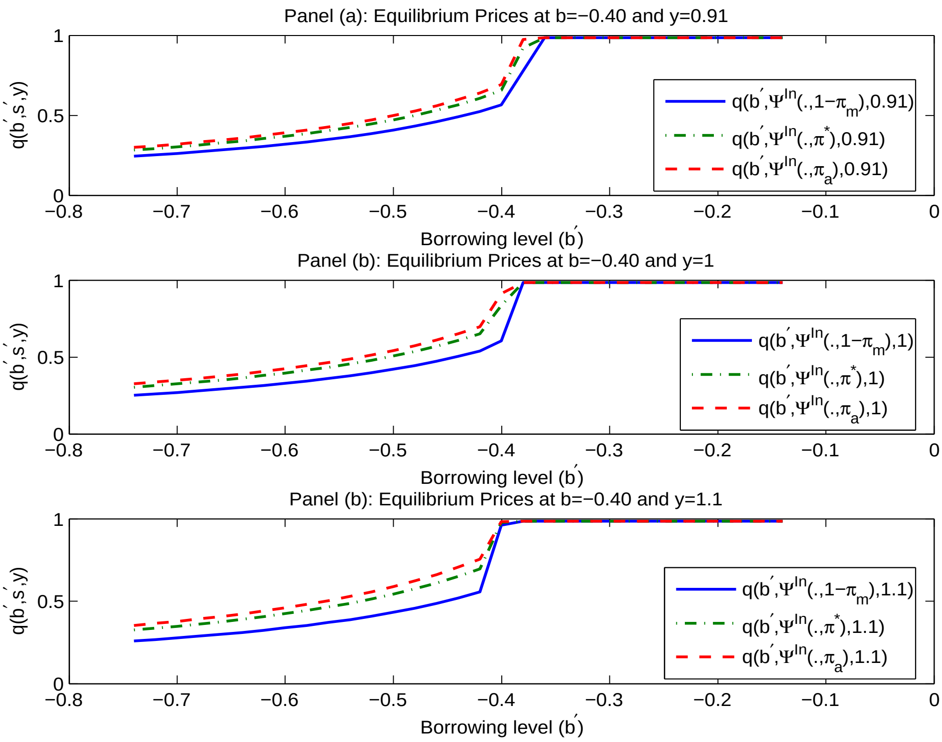
Figure 1.7: Negative Relation between Sovereign Ratings and Loan Prices



The model generates a relation between sovereign ratings and spreads that is consistent with the data. Higher ratings imply lower spreads and viceversa. This relation between sovereign ratings and bond prices is important in creating the incentives to sustain higher debt levels in equilibrium because $a - type$ governments are able to obtain a lower interest rate by signaling its type. Moreover, an $m - type$ government that mimics the $a - type$ government and increases its sovereign rating also faces a lower interest rate. Another important property of the equilibrium is that at a given s, if default is optimal for a government of type i at b^1 with $b^1 > b^0$, then default is also optimal for b^0. That is, $D_i(b^1, s) \subseteq D_i(b^0, s)$. This implies that $p(b^0, s, y) \geq p(b^1, s, y)$ and $q(b^0, s, y) \leq q(b^1, s, y)$.

For completeness, in Figure 1.8, I show the price schedule as a function of future debt levels when $b = -0.40$ for different income levels and sovereign ratings. Panel (a), (b) and (c) correspond to low ($y = 0.91$), average ($y = 1$) and high income ($y = 1.1$) respectively.

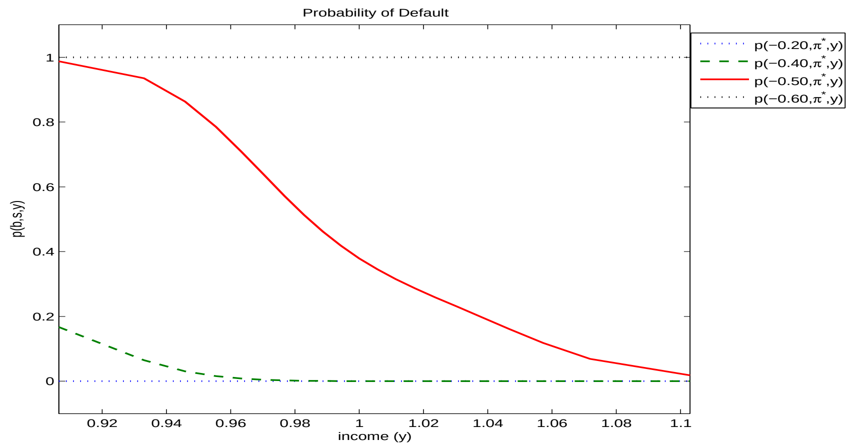
Figure 1.8: Price Function for different income and rating levels



Higher ratings have value because they affect the price at which the governments borrow. Within each panel, the price function is plotted for $s \in \{1 - \pi_m, \pi^*, \pi_a\}$ that correspond to the lowest, the average and the highest sovereign rating. The relation between sovereign ratings and interest rates is also evident from this picture. A higher rating implies a lower interest (whenever the interest rate is not equal to $(1+r)^{-1}$). From this figure we can see that as income increases the price for new loans also increases (interest rates decrease).

Default episodes observed during the last decade were associated with severe economic crisis. Figure 1.9 shows that in the model, default incentives are stronger during low income realizations. This figure displays the default probability over income levels for different combinations of debt, evaluated at average score, π^*. When debt is $b = -0.60$, the probability of default is equal to one for all income levels. At the average debt level $b = -0.40$ the default probability is strictly decreasing in y for $y \leq 1 = E[y]$. After that, the probability of default is equal to zero.

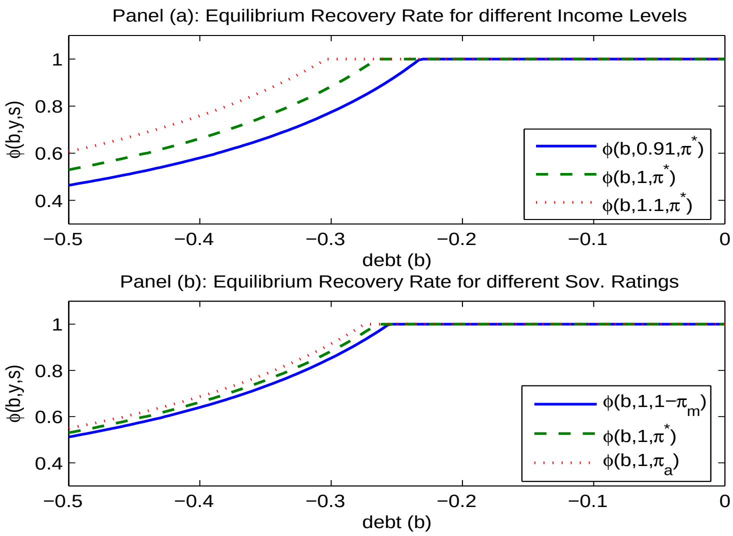
Figure 1.9: Default Incentives and Income



The introduction of endogenous recovery has important implications for resolving the sovereign debt puzzle. The country incentive to default depends on the renegotiation agreement on debt reduction. As in Yue (2007), it can be shown that for a given bargaining power θ, there exists a threshold $\bar{b}(y,s)$ such that the equilibrium recovery function ϕ satisfies

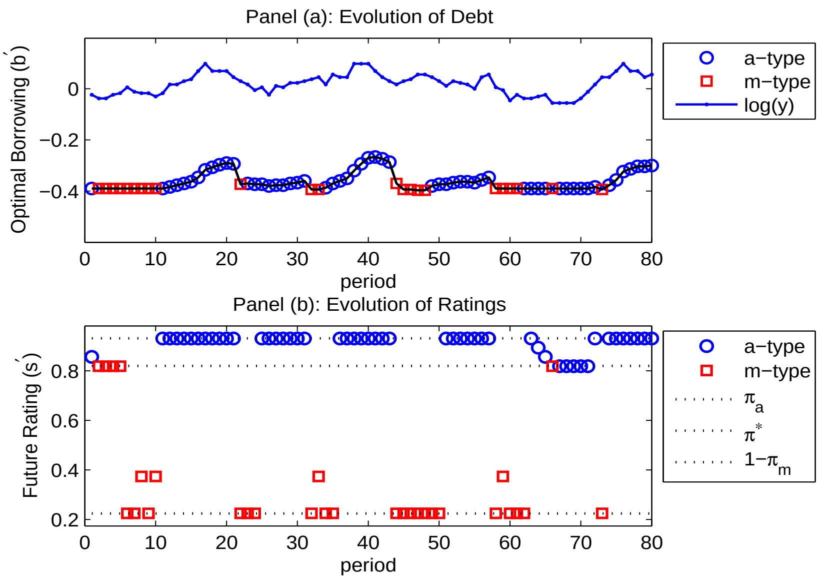
$$\phi(b,y,s) = \begin{cases} \frac{\bar{b}(y,s)}{b} & \text{if } b \leq \bar{b}(y,s), \\ 1 & \text{if } b > \bar{b}(y,s). \end{cases}$$

Figure 1.10: Recovery Schedule



Hence, debt recovery rates decrease with the amount of defaulted debt, and there is no debt reduction for debt levels smaller than the threshold. Observations from recent sovereign defaults are consistent with the equilibrium recovery schedule. Panel (a) of Figure 1.10 shows the equilibrium recovery schedule as a function of debt and income evaluated at π^*. The resulting recovery rate is increasing in the level of income. A novelty of this model is the relation between recovery rates and the sovereign rating. Panel (b) of Figure 1.10 displays the recovery rate evaluated at average income for different values of s. We can note that the recovery rate is increasing in s. In summary, the bargaining mechanism generates that repayment incentives are stronger in good times, debt reduction is lower as debt levels decrease and that countries that defaulted on a bigger debt stay longer in financial autarky. The average recovery rate is equal to 59%. The average recovery rate for emerging economies computed from Chuhan and Sturzenegger (2005) and Moody's (2006) report on recovery rates for defaults in the last decade is found to be 62.5%.

Figure 1.11: Evolution of Debt and Ratings



To conclude this section, in Figure 1.11, I show the evolution of debt and ratings in a simulation of 100 periods with no defaults. In Panel (a), circles correspond to the borrowing decision $b'_a(b, y, s)$ of the $a-type$ government. In Panel (b), circles correspond to the sovereign rating obtained by an $a-type$ government after the borrowing decision $s' = \Psi^{In}(b', d = 0, b, y, s)$. Similarly, in Panels (a) and (b), squares correspond to debt levels $b'_m(b, y, s)$ chosen by an $m-type$ government and ratings $s' = \Psi^{In}(b', d = 0, b, y, s)$ obtained by an $m-type$ government respectively. From Panel (a), we observe that the $m-type$ government generally borrows more than the $a-type$ government. Panel (b) show how fast lenders learn about the government type. The process for types implies that the score will tend to return to the mean π^* if no information can be extracted from the government actions, i.e. $\sigma^{In}(b', d = 0, b, y, s) = s$. Cycles of signaling and confusion alternate over time.

1.6 The Debt to Output Puzzle

Recent models of sovereign debt explained how the risk of default interacts with business cycles in emerging economies (see Aguiar and Gopinath (2006), Arellano (2007) and Yue(2007) for example). However, at the observed default frequency, the level of debt to output ratio generated by these models was much lower than what we observe in the data. In particular, the debt to output ($\frac{b}{y}$) ratio[13] was found to be 18% in Aguiar and Gopinath (2006), 7.33% in Arellano (2007) and 9.69% in Yue (2007).

Table 1.6: Main Result

	Model	Aguiar and Gopinath	Arellano	Yue
Debt to Output Ratio	39.6 %	18.0 %	7.3 %	9.7 %
% of data	45 %	20 %	8 %	11 %

The debt to output ratio computed at the estimated equilibrium is 39.6%, that is the model generates more than five times the debt to output ratio generated by Arellano (2007), more than three times of the value found by Yue (2007) and more than twice the ratio obtained by Aguiar and Gopinath (2006). Note that the debt to output ratio was not part of the set of moments used in the estimation. When computed from the the Global Development Finance *II* data set (compiled by the World Bank) the average ratio of Government Debt to annual GNP in Argentina since 1970 is 88%. The aim of this paper was to close the gap between the data and

[13]These models were also quarterly models and were calibrated to the Argentinean economy.

the existing literature by introducing political uncertainty in a standard small open economy model. Effectively, when computed as a fraction of annual output, the the debt to output ratio in the model is close to 45% of that in the data.

The information structure and the endogenous determination of default penalties (recovery rates and periods of exclusion) are the keys to understanding the intuition behind the main result. In the next section, I will show how the decisions of the government affect its reputation in credit markets and hence the price of new loans. In summary, a patient government may be able to get lower interest rates by choosing not to default and signal its type during bad times of the economy. Similarly, if the government has an unresolved default the aligned type is willing to repay sooner than the misaligned type to go back to financial markets with a higher reputation. Moreover, at some rating levels, a less patient government can mimic the behavior of the patient government and borrow at low interest rates. All these factors combine to generate the necessary incentives to make borrowing more attractive than defaulting. Furthermore, the introduction of endogenous recovery rates and periods of exclusion makes transiting through a default state much more costly for any government type and it has two effects over loan prices. First, at a given debt level, interest rates decrease because default probabilities are lower than in a world with costless return to markets. Second, the introduction of endogenous recovery rates makes borrowing more appealing because governments are able to obtain contingent repayments after a default.

The estimated parameters are in line with those used in previous studies of sovereign debt. In this paper, the aligned government discounts the future at a rate equal to 0.985 and the misaligned government discounts the future at a rate equal to 0.805 ($= .818\beta$). The discount factors used in the studies of Arellano (2007), Aguiar and Gopinath (2006) and Yue (2007) are 0.953, 0.80 and 0.74 respectively. Arellano (2007) and Aguiar and Gopinath (2006) do not consider the possibility of renegotiation and periods of exclusion are exogenous. The calibrated probability of returning to capital markets after a default implies that countries spend approximately 3.84 quarters in autarky for the case of Arellano (2007) and 2.5 years in the case of Aguiar and Gopinath (2006). Yue (2007) considers debt renegotiation and also finds that Argentina has a higher market power (equal to 0.83) than lenders at the renegotiation stage. However, that model implies that countries stay in financial autarky on average only one quarter, a period much shorter than that observed for emerging economies. The estimation of the benchmark model developed in this paper implies that the country stays in financial autarky for about 4 quarters and the bargaining power of Argentina is 0.574.

In Section 1.7, I show that the combination of private information and endogenous recovery rates are essential ingredients to sustain the observed debt to output ratios. A model with full information and zero recovery rates generates an equilibrium debt to quarterly output ratio that is only 13%. In addition, a model with full information and endogenous recovery generates a debt to quarterly output ratio that is still only 28%.

1.7 Private Information is Important

To be able to disentangle the effects of private information and endogenous recovery rates, in this section, I explore how the main results would change if two extensions of the benchmark model were considered. Both extensions share the characteristic that government types are public information. That is, in this environment, foreign lenders are able to observe the government type and the type changes over time. The first case corresponds to a model were recovery rates are zero and return to markets is exogenously given (as in Arellano (2007) and Aguiar and Gopinath (2006)). After a default countries are able to borrow again in the following period with probability α and with probability $(1-\alpha)$ they stay one more period in financial autarky. In the second case, I consider a model also with full information but with endogenous recovery rates (as in Yue (2007)). In both cases I estimate the parameters of the models to match the moments used in the benchmark case. The equilibrium price function and the renegotiated repayment fraction will depend on the government type and not in the sovereign rating. In fact, the sovereign rating does not have any meaning in a context with full information.

1.7.1 Full Information Models: Estimation and Results

The risk free rate, the parameters of the endowment process and the output cost are set to the same values of the benchmark model. The parameters left are estimated through Simulated Method of Moments as described before. The set of moments used are also the same used for the estimation of benchmark model and depicted in Table 1.5. For the case of zero recovery rates the set of parameters to define are $\{\delta_m, \pi_a, \pi_m, \alpha\}$. The value of α is chosen in order to obtain an average period of financial exclusion consistent with the data. For the case of endogenous recovery rates the set of parameters to define are $\{\delta_m, \pi_a, \pi_m, \theta\}$. These parameters are estimated by matching the default probability, the standard deviation of the current account to output ratio, the mean recovery rate and the average period of financial exclusion.

The discount factor of the misaligned government is found to be similar in the three models. The model with full information and endogenous recovery rates predicts that the country will have an aligned government around 80%. For the case of the model with zero recovery rates this value is approximately 74%. The bargaining power of the government do not present major differences across the models either.

Table 1.7 displays the parameters values and the moments generated by the models.

Table 1.7: Effects of Private Information and Recovery Rates

	Benchmark	Full Info Zero Recovery	Full Info Endog. Recovery
Parameters			
δ_m	0.818	0.856	0.827
π_a	0.941	0.880	0.914
π_m	0.759	0.782	0.782
θ	0.574	–	0.605
α	–	0.231	–
Moments			
Default probability	0.65	0.51	0.59
Std Dev CA/Y	1.46	1.25	1.52
Average exclusion period	4.07	4.33	2.3
$\sigma(c)/\sigma(y)$	1.16	1.02	1.13
Debt to Output ratio	39.5	13.4	28.4

As it is clear from the last row of Table 1.7, the debt to output ratios that models with full information predict are much lower than in the model with private information. The model with zero recovery rates generates a debt to output ratio that is one third of that in the benchmark model. Including endogenous recovery rates and periods of exclusion generates a higher debt to output ratio but still only 72% of the equilibrium ratio in the presence of private information.

When the political states are observable, reputation has no value. The main motivation for the borrowing and default decisions is consumption smoothing. In that case, the misaligned government will tend to default more often and because government types are known it will be charged a much higher interest rate than the aligned government. Moreover, for the aligned government it is not necessary to change its behavior in order to signal its type. This lowers the benefits from not defaulting in bad times. The effects of private information and recovery rates can be seen clearer in Figures 1.12 and 1.13. In these Figures, I plot the price function in a model with full information and no recovery rates $q^{FINR}(b', i, y)$, the price function for the model with full information and endogenous recovery rates and periods of exclusion, $q^{FINR}(b', i, y)$, and the price function for the benchmark model $q(b', s', y)$. In a model with no recovery rates there is a strong relation between the default probability and the price of the bonds. The introduction of recovery rates reduces the losses associated with a default and lowers interest rates. The introduction of private information makes incentives not to default stronger shifting the price schedule to towards higher levels of debt.

Figure 1.12: Price Functions for different models

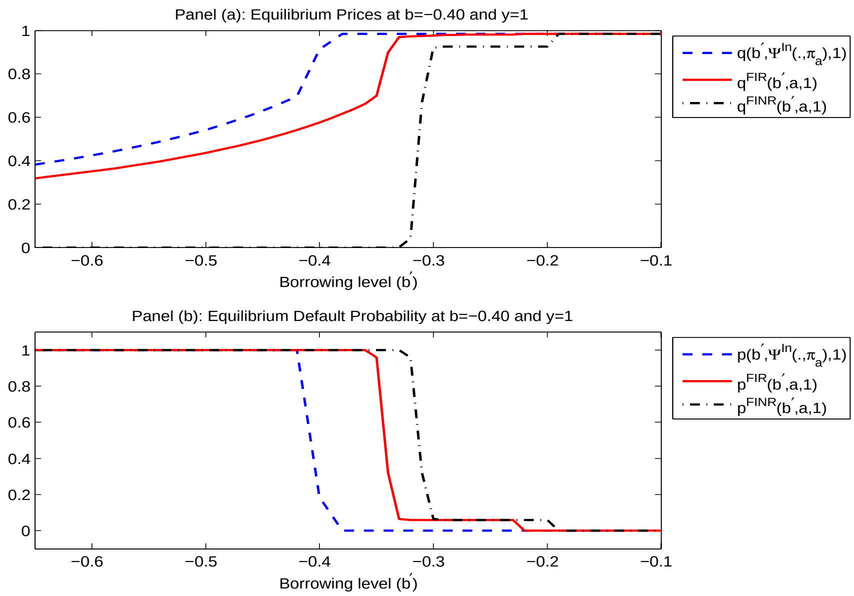
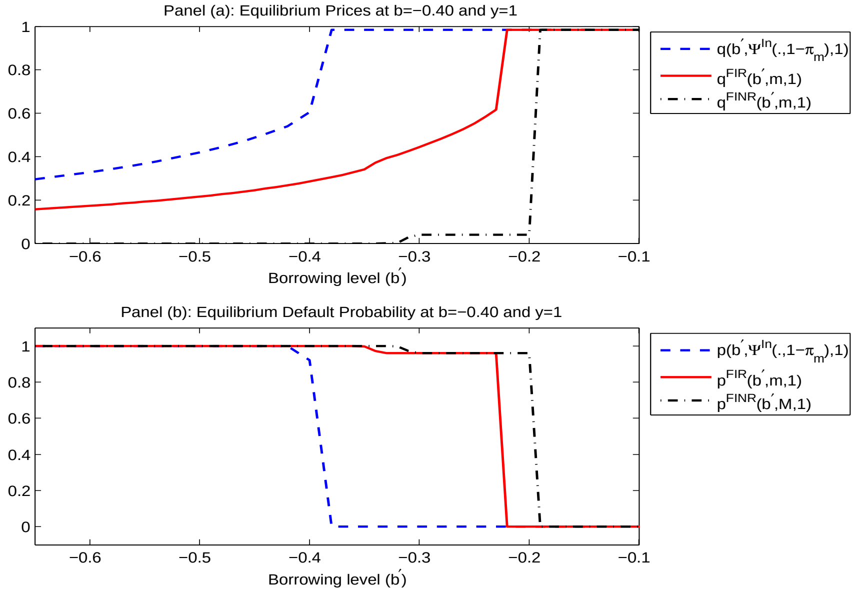


Figure 1.13: Price Functions for different models



1.8 Comparison to the Data: Business Cycle moments

The business cycle moments for emerging economies are well documented and the results obtained in this paper are consistent with previous studies.[14]

Using data from Argentina, I found that consumption and output are highly correlated ($corr(y, c) = 0.97$) and the correlation between the current account to output ratio, ca/y and output is -0.82. The data on output, consumption and current account is taken from the Ministry of Economics in Argentina for the period 1980q1 to 2006q4. Variables are real and seasonally adjusted. Output and consumption are detrended using the HP filter with parameter 1600. The data on spreads is computed from J.P. Morgan's Emerging Markets Bond Index (EMBI+) for Argentina from 1993 to 2001. The sovereign spread is the difference between the Argentinean interest rate and the real return on a U.S. Treasury bill of similar

[14] See Neumeyer and Perri (2005)

maturity. The standard deviation of spreads is 3.07% and the average spread is 7.75%. If the statistics on spreads are derived from 3-year foreign currency denominated bonds from 1993 to 2001 taken from Broner, Lorenzoni and Schmukler (2004) the value of the standard deviation is 1.69% and the average spread is 4.08%. The rating data is computed using the Standard & Poor's rating for Argentina since its first date available, 1993q3, to 2006q4. The correlation of ratings and output is 47% and the correlation with spreads is -81%.

Table 1.8 displays the moments from the data and those from the simulated model. The model generates business cycle statistics that are in line with the data. In particular, consumption is highly correlated with output and the current account is countercyclical.

Table 1.8: Non-Target Moments

Moment	Data	Model	Arellano (2007)	Aguiar & Gopinath (2006)	Yue (2007)
Corr(y, c) (%)	97	93	91	97	n.a.
Corr($ca/y, y$) (%)	-82	-11	-15	-12	-14
EMBI Std. Dev. (%)	3.07	1.95	10.6	0.12	1.32
EMBI Mean Spread (%)	7.75	1.98	10.4	0.57*	1.84
Std. Dev. short-term spread(%)	1.68	1.95	10.6	0.12	1.32
Mean short-term spread (%)	4.08	1.98	10.4	0.57*	1.84
Corr($y, spread$) (%)	-76	-35	-22	-2	-18
Corr($ca/y, spread$) (%)	75	32	17	38	54
Corr($y, rating$) (%)	47	21	n.a.	n.a.	n.a.
Corr($spread, rating$) (%)	-81	-66	n.a.	n.a.	n.a.

*Max value is reported only

The model delivers a standard deviation of spreads that is consistent with short-term bond data but a lower mean spread (even when compared with short-term bonds). However, this anomaly of the model has been present in previous studies as well. Arellano (2007) obtains a much higher mean spread but at the cost of a standard deviation that is more than 3 times of that observed in the data. Aguiar and Gopinath (2006) only report the maximum value of spread recorded in their simulation and even so this value is only 13% of the mean of short term spreads and 7% of the average EMBI+. Yue (2007) reports mean spreads than are also lower than in the data.

The benchmark model is able to capture the countercyclical behavior of interest rates and the positive correlation between the current account and spreads. In particular, the correlation of income and spreads is -35 percent and the correlation between the spreads and the current account is 32 percent. A new dimension considered in this paper is the relation between interest rates, sovereign ratings and output. Consistent with the observations of the Argentinean economy in the past decade, ratings are positively correlated with output (corr($y, rating$) = 21 percent) and present a high negative correlation with spreads (corr($spread, rating$) $= -66$ percent).

In summary, the main regularities of emerging economies are that interest rates are countercyclical, the current account is countercyclical and interest rates and the current account are positively correlated. The model in this paper matches the data in that it simultaneously delivers a higher volatility of consumption relative to output, countercyclical interest rates and a countercyclical current account. It also displays a positive correlation between interest rates and the current account.

1.9 Conclusion

The average country, among emerging economies has experienced 3 defaults every 100 years, and sustains an average external debt to output ratio around 58%. Previous models of sovereign debt are unable to account jointly for the debt to output ratio and the observed default frequency. The combination of government reputation with endogenous periods of exclusion and debt renegotiation produce a debt to output ratio at least 100% higher than in previous models and accounts for 50% of the ratio observed in the data.

The information structure and the endogenous determination of default penalties, including recovery rates and periods of exclusion, are essential ingredients to obtain the main result. In section 1.6, I show that a model with full information and zero recovery rates generates an equilibrium debt to output ratio that is only 34% of the benchmark model. In addition, a model with full information and endogenous recovery generates a debt to output ratio that is 72% of the model with private information. In these models, reputation has no value because political states are observable, thus a repayment decision per se has no effect on future prices.

In section 1.8, I show that the model account for the main regularities of emerging economies such as a higher volatility of consumption relative to output, countercyclical interest rates and a countercyclical current account. It also displays a positive correlation between interest rates and the current account.

In this paper, I show that a model of reputation and endogenous recovery rates accounts for a large fraction of the debt to output ratio observed in the data. However, other puzzles are still present. The main anomaly of the paper is the low average interest rate (and spreads) generated in equilibrium. From Table 1.8 it is evident that previous studies are also unable to generate the observed mean spread. Other channels beyond the one identified here could affect the borrowing decisions of the government and lenders' expectations of the risk of default. The exploration of currency risk or debt with different maturities in a reputational model are left for future research.

Chapter 2

Politico Economic Consequences of Rising Wage Inequality[1]

2.1 Introduction

In this chapter we ask whether the observed increase in wage inequality and the decrease in median to mean wages can explain some part of the increase in transfers to low earnings quintiles and increase in effective tax rates for high earnings quintiles in the U.S. over the past few decades. To answer this question we use a model with uninsurable, idiosyncratic shocks to labor efficiency similar to Aiyagari [5]. With incomplete markets, the rising wage dispersion generates more individual consumption dispersion and an increased role for government insurance (transfer) programs. The benefits of such transfer programs may be offset by the costs associated with financing through distortionary taxation. We use a political recursive competitive equilibrium concept pioneered in Krusell, et. al. [73]. Specifically, political outcomes are endogenously determined by a median voter who chooses a proportional

[1]This chapter borrows extensively from a joint project with P. Dean Corbae and Burhanettin Kuruscu (see Corbae, D'Erasmo and Kuruscu [40]).

tax rate that is required to be consistent with a sequential equilibrium of a competitive economy. Obviously, the difficulty in the analysis arises out of the fact that the endogenous policy outcomes and the endogenous evolution of the wealth distribution are interconnected. Idiosyncratic uncertainty greatly complicates the determination of the median voter.

The specific experiment we consider is to choose a transition matrix to match observed mobility in wages between 1978 to 1979 in the PSID dataset and show that these numbers are consistent with "low" inequality. Then we reparameterize the transition matrix to match the observed mobility between 1995 to 1996 and show that these numbers are consistent with "high" inequality. Then we ask what proportional tax rates the median voter would choose for each of the two parameterizations. At this new tax rate, we compute the changes in effective tax rates by quintile (normalized by the middle quintile). Since during the 1979 to 1996 period the wage data was also characterized by a sustained decrease in the median to mean wage, there are potentially important differences between proportional taxes chosen by a median voter and a utilitarian planner. We find that in general the results from the median voter model are closer to the data than those chosen from a utilitarian mechanism.

The main difference from previous work in this area is the introduction of idiosyncratic uncertainty in a political-economy model.[2] For instance, what many consider to be the canonical political economy model by Krusell and Rios-Rull [74] assumes that households are heterogeneous in their earnings but there are complete markets so that there is no uncertainty in the present discounted value of earnings. Complete markets also implies that the differences in initial wealth between households persist indefinitely (i.e. it is possible to choose an exogenous initial wealth

[2] There are several papers which consider a social planner's utilitarian choice of exogenous taxes with incomplete markets and idiosyncratic uncertainty. See for example, Aiyagari [6] and Domeij and Heathcote [44].

distribution that is consistent with a steady state which replicates itself every period from $t = 0$) which allows them to identify the median voter ex-ante. In a related paper by Azzimonti et. al. [15], the authors use a first-order approach and show that aggregate state can be summarized by the mean and median capital holdings in a model without uncertainty. They also include a proof that their environment yields single-peaked preferences. The closest paper to ours is Aiyagari and Peled [7]. They consider a model with idiosyncratic uncertainty, however they restrict off-the-equilibrium path beliefs to be those from the steady state rather than sequentially rational beliefs.

The paper is organized as follows. The data facts are presented in section 2. The model is presented in section 3. In section 4, we discuss how we calibrate the benchmark model. In section 5 we present a quantitative experiment to study the effect of the increase in earnings volatility on tax choices. Finally, in Section 6 we conduct a welfare analysis.

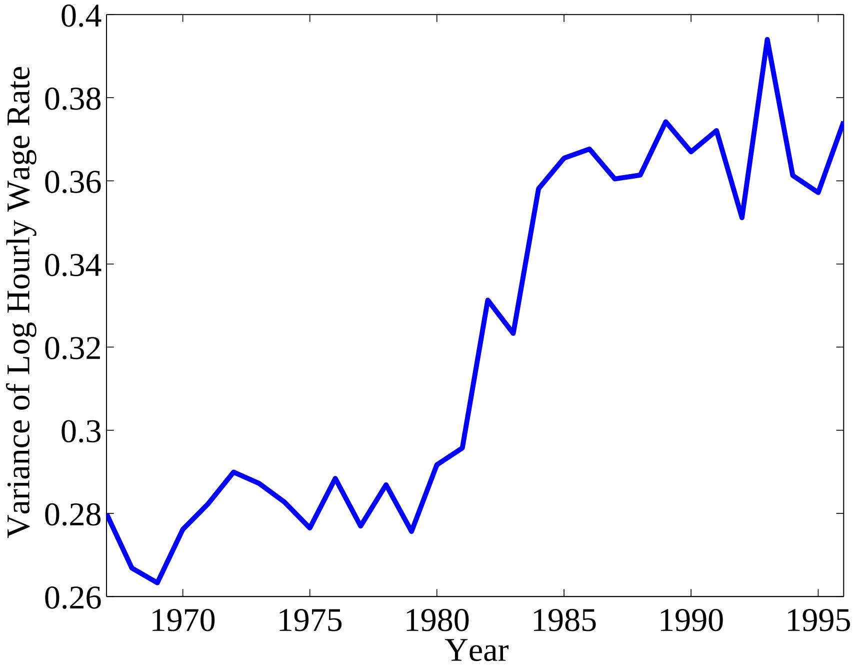
2.2 Data Facts

It is well documented that there has been an increase in wage inequality during the past three decades. Using the Panel Study of Income Dynamics, in Figure 1 and 2 we document a substantial increase in the variance of the log-wage as well as a decline in the median to mean ratio of wages for heads of households between 20 and 59 and who work for no less than 520 hours (see our Data Section for a complete description of the selection criterion we use).[3] We choose this selection criterion because we will work with a infinitely lived agent model.[4] There appear to be two

[3]There are many papers documenting the rise in wage inequality. See, for example, Autor, et. al. [14] and Heathcote, et. al. [62].

[4]As part of a sensitivity analysis, we plan to relax the restriction that heads of households work for no less than 520 hours. This selection criterion rules out people who are unemployed for long durations, those out of the labor force, and some students.

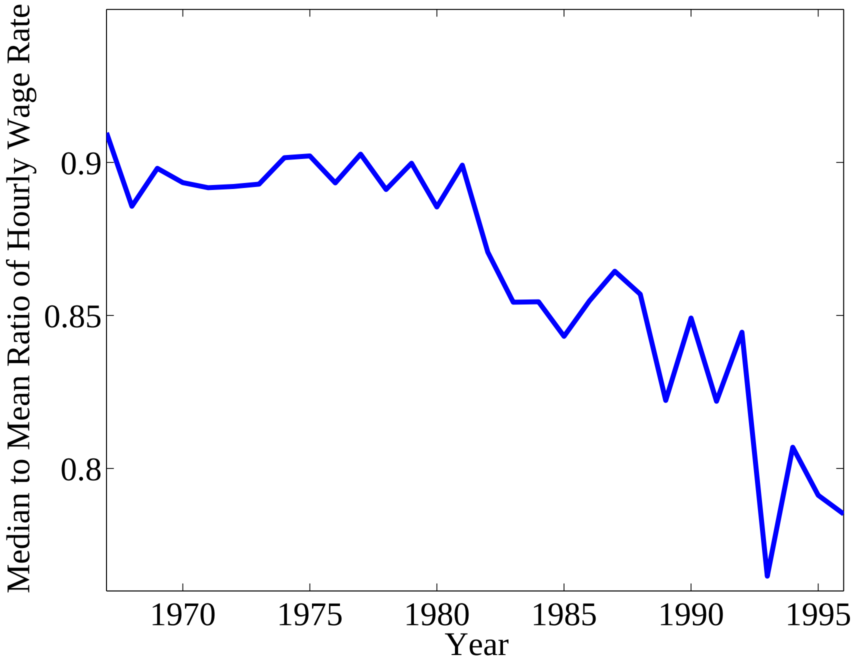
different regimes in Figure 1; one with low variance until the beginning of the 80's where the mean variance of log wages is around 30% and another regime with high variance from the mid 80's to 1996 with mean variance approximately equal to 39% (an increase of more than 30%). From Figure 2, we observe that during the same period the median to mean ratio displayed a sharp decrease of around 10%.[5]



Source: Panel Study of Income Dynamics.

Figure 2.1: Increase in wage inequality $1966 - 1996$.

[5]We consider 1996 as the second regime date since that year is the last year for which the PSID provides annual data. Specifically, after 1996, the PSID provides biannual data. Since our model will be annual, calculations based on two year mobility matrices would underestimate risk.



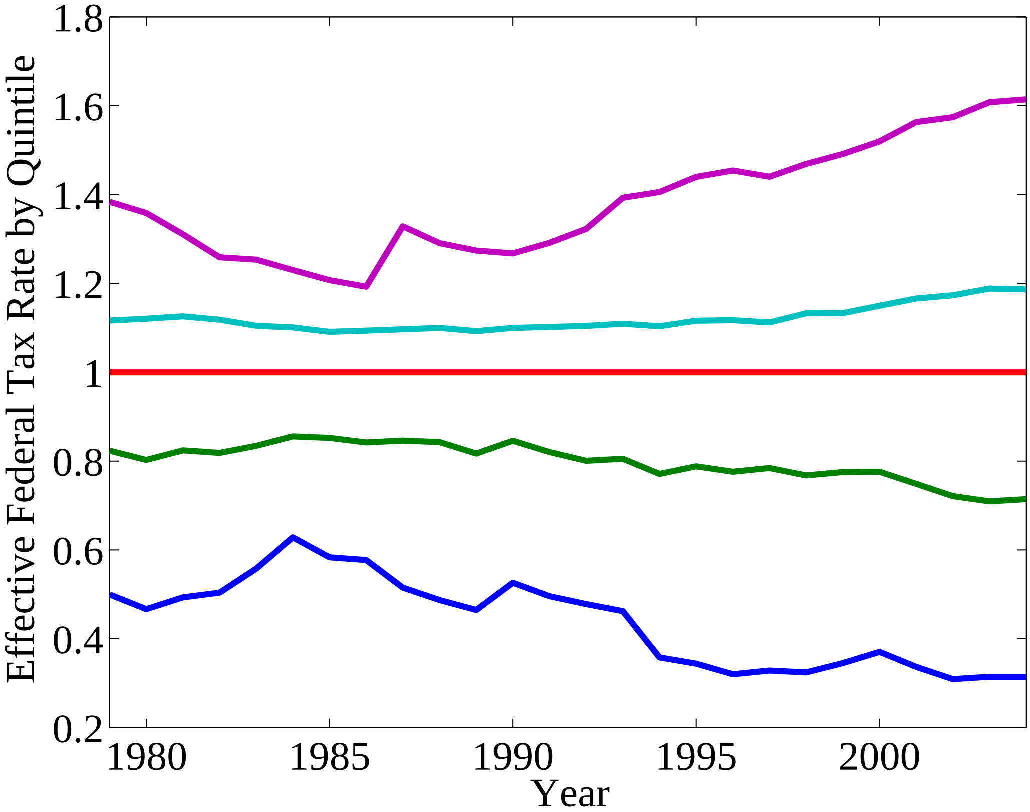
Source: Panel Study of Income Dynamics.

Figure 2.2: Decrease in median to mean ratio of log wages $1967 - 1996$.

The Congressional Budget Office (CBO) recently published data on effective federal tax rates in the United States for the past two and a half decades. Given we are focusing on wages for households between 20 and 59, we consider effective federal tax rates for the entire population less elderly (defined as having at least one head over the age of 65 and no children under 18).[6] The federal effective tax rate is the sum of all tax types paid by households. The effective tax rate is defined to be the tax liability of a household divided by its post transfer (but pre-tax) income, which we will denote I_t. It is comprised of effective individual income tax rates, effective social insurance taxes, effective corporate income taxes, and effective excise taxes.

[6]Again see our Data Section for a complete description of the data and the selection criterion we use.

One of the important facts that we observe is that redistribution through the tax system in the U.S. has increased after the 1980's. Figure 3 illustrates the effective tax rates paid by each income quintile (normalized by the effective tax rate paid by the middle income quintile). It is clear from the figure that while the effective tax rate for the higher income quintiles increased relative to that of the middle quintile, the effective tax paid for the lower income quintiles declined relative to that of the middle quintile. For example, the effective tax rate for the highest quintile rose from around 1.38 times the value of that payed by the middle quintile in 1979 to around 1.45 times it in 1996 (an increase of 5%). At the same time the relative effective tax rate for the lowest quintile decreased by more than 35% (from 0.5 times the value of that payed by the middle quintile to 0.32 times it).



Source: Congressional Budget Office.

Figure 2.3: Effective Federal Tax Rate by quintiles 1979 – 2004.

The CBO also provides data on before-tax and after-tax income for each income quintile. As an alternative measure of redistribution, we note that pre-tax income inequality between quintiles (i.e. variance of log pre-tax income) increased by 21.02 log points from 1979 to 1996 while after-tax income inequality increased by 15.87 log points over that same period.

The relative changes in effective taxes by each quintile we see in Figure 3 could be due to several reasons. First, for given income levels, changes in the tax code may create more redistribution. Second, for a given tax rate schedule, increases in income inequality can generate more redistribution since the tax system is progressive. For example, increases in income of higher quintiles could generate increases in effective taxes because people in those quintiles move up the tax schedule facing higher marginal tax rates. The opposite could happen if lower quintiles experience declines in their income; they move down the tax schedule and face lower marginal tax rates.

Since effective federal income taxes make up the largest percentage (at least half) of effective federal taxes, in order to gain some insight into how much the changes in effective taxes in Figure 3 are due to income changes versus changes in the tax code, we use a decomposition of effective federal income tax rates from a paper by Harris, et. al. [59].[7] Specifically, the authors decompose the change in effective income taxes for all households into changes due to the change in the tax code and changes due to other factors such as income and demographics. To understand how much of the redistribution we see in Figure 3 is due to changes in the tax code versus income changes, we use their data in the following way. We calculate the effective income tax from 1979 to 2000 due solely to changes in income, given estimates in Harris, et. al. [59].[8] Figure 2.4 illustrates the normalized actual effective income taxes (solid red line) and normalized income taxes that would arise due only to changes in income (dashed black line). As evident in Figure 2.4, the changes in effective taxes due only to changes in income are rather small and most of the widening seems to be due to changes in the tax code.

[7] To see that effective income taxes compose the largest percentage of total effective taxes, see Table 1A in http://www.cbo.gov/ftpdocs/77xx/doc7718/SupplementalTables.xls.

[8] Specifically, $\Delta ei^q_{79,00}(Income)$ is simply the sum of the column entitled "All Income Adjustments" in Table 4 of Harris, et. al. [59]

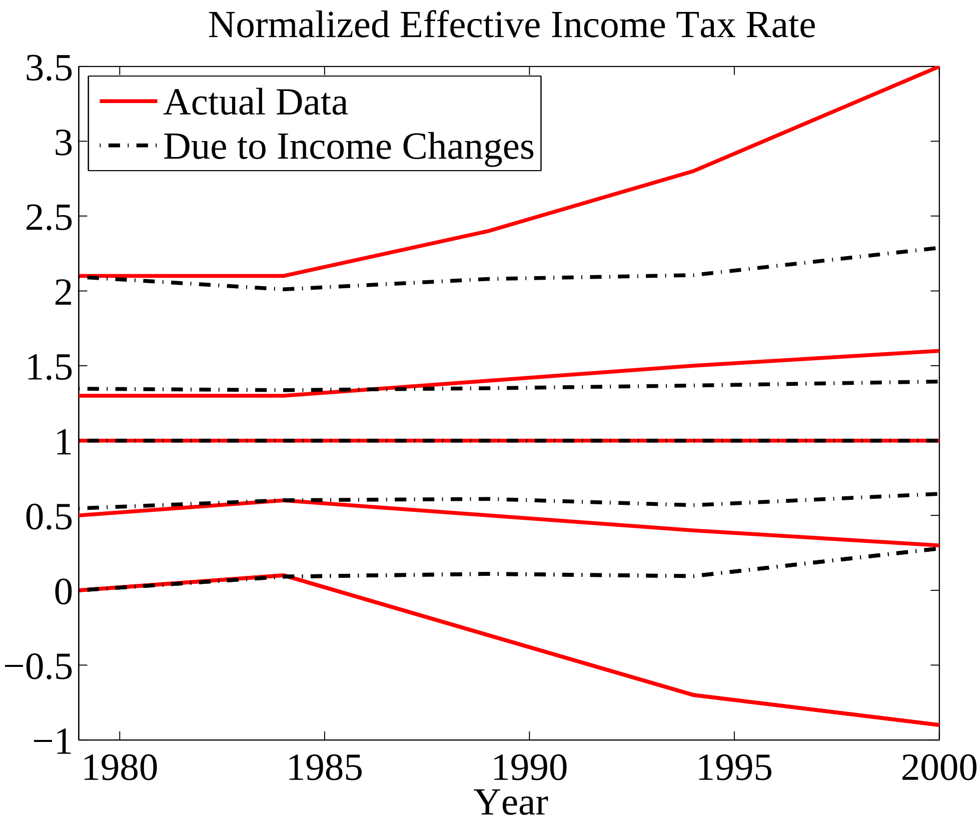


Figure 2.4: Decomposition of Effective Income Tax Rate by quintiles 1979 – 2000.

In summary, as is clear from Figures 1 through 4, changes in wage inequality may have important implications for changes in effective tax rates as part of a redistributive or social insurance mechanism. We now turn to a simple incomplete markets model Aiyagari [5] where there is a role for redistribution to illustrate this mechanism.

2.3 Model

2.3.1 Environment

There is a unit measure of infinitely-lived households. Their preferences are given by:

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t, n_t)\right] \tag{2.1}$$

where c_t denotes consumption, $n_t \in [0,1]$ denotes labor supply in period t, and $\beta \in (0,1)$ is the discount factor. We assume that the period utility function has the form introduced by Greenwood, Hercowitz and Huffman [56]:

$$u(c_t, n_t) = \frac{1}{1-\gamma}\Big[c_t - \chi\frac{n_t^{1+1/\nu}}{1+1/\nu}\Big]^{1-\gamma} \tag{2.2}$$

where γ is the coefficient of relative risk aversion and ν is the intertemporal (Frisch) elasticity of labor supply.

Production takes place with a constant return to scale function, whose inputs are capital and labor

$$Y_t = F(K_t, N_t) = K_t^{\alpha}N_t^{1-\alpha} \tag{2.3}$$

where capital letters denote aggregates. The final good can be used for consumption or investment. Capital depreciates at rate δ.

Each household faces an uninsurable, idiosyncratic labor efficiency shock $\epsilon_t \in E$ which evolves according to a finite state markov process $\Pi(\epsilon_{t+1} = \epsilon\prime|\epsilon_t = \epsilon)$. Household earnings are given by $w_t\epsilon_t$ where w_t is a competitively determined wage. An individual household can self insure by holding k_t units of capital which pays a risk free rate of return r_t. Households are allowed to borrow up to an exogenous borrowing limit b. For simplicity, we assume that the interest paid on borrowings are tax deductible.

The government taxes household capital holdings and labor income at the same proportional rate denoted τ_t, spends G_t and provides lump-sum transfers denoted T_t. The government is assumed to run a balanced budget so that

$$G_t + T_t = \tau_t \left[r_t K_t + w_t N_t \right]. \tag{2.4}$$

2.3.2 Recursive Competitive Equilibrium

Let the joint distribution of capital and efficiency levels across households be denoted $\Gamma_t(k_t, \epsilon_t)$ with law of motion $\Gamma_{t+1} = H(\Gamma_t, \tau_t)$.[9] Then the aggregate capital stock is given by

$$K_t = \int k_t \; d\Gamma_t(k_t, \epsilon_t) \tag{2.5}$$

and aggregate labor is given by

$$N_t = \int \epsilon_t \; n_t d\Gamma_t(k_t, \epsilon_t). \tag{2.6}$$

Perfect competition in factor markets implies

$$\begin{aligned} r_t &= \alpha K_t^{\alpha-1} N_t^{1-\alpha} - \delta \\ w_t &= (1-\alpha) K_t^{\alpha} N_t^{-\alpha}. \end{aligned} \tag{2.7}$$

The economy-wide resource constraint in each period is given by

$$C_t + G_t + K_{t+1} = Y_t + (1-\delta) K_t \tag{2.8}$$

Letting x denote x_t and x' denote x_{t+1}, we can write the household problem

[9] Since there are no other assets besides capital, the distribution of capital and the distribution of wealth are identical. We will use these definitions interchangeably.

recursively as[10]

$$V(k,\epsilon;\Gamma,\tau) = \max_{c,n,k'} u(c,n) + \beta \sum_{\epsilon'} \Pi(\epsilon'|\epsilon) V(k',\epsilon';\Gamma',\tau') \tag{2.11}$$

s.t.

$$\begin{aligned} c + k' &= k + \left[r(K,N)k + w(K,N)\epsilon n\right](1-\tau) + T \\ k' &\geq -b \\ \Gamma' &= H(\Gamma,\tau) \\ \tau' &= \Psi(\Gamma,\tau) \end{aligned}$$

where the perceived law of motion of taxes is given by $\tau_{t+1} = \Psi(\Gamma_t, \tau_t)$. The solution to the individual's problem generates decision rules which we denote

[10]The utility function given in equation (2.2) has the convenient property that the labor supply choice is independent of the consumption-savings choice. In particular, assuming an interior solution, individual labor supply is a simple function of the after-tax labor income:

$$n = \Big[\frac{w\epsilon(1-\tau)}{\chi}\Big]^{\nu} \tag{2.9}$$

It is important to note that the optimal labor supply does not depend on household wealth. This property has the useful implication that equilibrium aggregate effective labor supply depends only on the inherited aggregate capital stock, the current tax rate, and the time-invariant distribution over the set of productivity shocks:

$$N = \Big[\sum_i \pi_i^* \epsilon_i^{1+\nu} \Big(\frac{(1-\tau)(1-\alpha)K^{\alpha}}{\chi}\Big)^{\nu}\Big]^{\frac{1}{1+\alpha\nu}}. \tag{2.10}$$

This simplifies the solution of our problem because equilibrium prices become a function of the aggregate capital stock and tax rates only (as before). With general preferences we would need another state variable - see appendix B in Krusell and Smith [75] for that case.

$$n = \eta(k, \epsilon; \Gamma, \tau),$$

$$c = g(k, \epsilon; \Gamma, \tau) \quad and$$

$$k' = h(k, \epsilon; \Gamma, \tau).$$

Before moving to the endogenous determination of tax rates via majority voting, it is useful to state a competitive equilibrium taking as given the law of motion of taxes.

Definition (RCE). Given $\Psi(\Gamma, \tau)$, a **Recursive Competitive Equilibrium** is a set of functions $\{V, \eta, g, h, \Gamma, H, r, w, T\}$ such that:

(i) Given (Γ, τ, H, Ψ), the functions $V(\cdot), \eta(\cdot), g(\cdot)$ and $h(\cdot)$ solve the hh's problem in (2.11);

(ii) Prices are competitively determined (2.7);

(iii) The resource constraint is satisfied

$$K' = K^{\alpha} N^{1-\alpha} + (1-\delta)K - \int g(k, \epsilon; \Gamma, \tau) d\Gamma(k, \epsilon) - G$$

where K and N are defined as in (2.5) and (2.6);

(iv) The government budget constraint (2.4) is satisfied

(v) $H(\Gamma, \tau)$ is given by

$$\Gamma'(k', \epsilon') = \int 1_{\{h(k,\epsilon;\Gamma,\tau)=k'\}} \Pi(\epsilon'|\epsilon) d\Gamma(k, \epsilon).$$

2.3.3 Politico Economic Recursive Competitive Equilibrium

In this section, we endogenize the tax choice. In particular, we allow households to vote on next period's tax rate τ'. Given that households are rational, a decisive voter evaluates the equilibrium effects of her choice, calculates the expected discounted utility associated with each τ', and chooses the tax rate which gives her highest utility. Since the source of household heterogeneity arises from the idiosyncratic shocks to earnings, we do not know who the median voter is as in the papers of, for instance, Krusell and Rios-Rull [74], we follow an alternative approach.[11] From each household choice we generate the distribution of "most preferred" tax rates and provided each household's derived utility is single-peaked, the median of the most preferred tax rates is chosen (i.e. it is the Condorcet winner which beats any alternative tax rate in a pairwise comparison). In this case, what the literature usually calls the median voter corresponds to the agent with capital holdings and productivity level that optimally chooses the median tax rate. It is important to appreciate that in environments with idiosyncratic uncertainty the median voter, in general, does not correspond to the agent with median capital holdings or median productivity shock.

To choose the most preferred tax rate, the household must choose among alternatives. Suppose that the household starts with state vector as before $(k, \epsilon, \Gamma, \tau)$ and consider a one period deviation for next period's tax rate to τ' not necessarily given by $\tau' = \Psi(\Gamma, \tau)$ while taking as given that all future $(t+2)$ tax choices will be given by the function Ψ. In that case, the household's problem is given by

$$\widetilde{V}(k, \epsilon, \Gamma, \tau, \tau') = \max_{c,n,k'} \; u(c, n) + \beta E_{\epsilon'|\epsilon} \left[V(k', \epsilon', \Gamma', \tau') \right] \tag{2.12}$$

[11]Only in the case of idiosyncratic transitory efficiency shocks are total resources, $(1 + r(1 - \tau))k + w\epsilon(1 - \tau) + T$, sufficient to know who the median voter is.

s.t.

$$
\begin{aligned}
c + k' &= k + [r(K,N)k + w(K,N)\epsilon n](1-\tau) + T \\
k' &\geq -b \\
\Gamma' &= \widetilde{H}\left(\Gamma, \tau, \tau'\right)
\end{aligned}
$$

where $\tilde{H}$ denotes the law of motion for Γ induced by the deviation, while all future distributions evolve according to H. Note that the future value function V is given by the solution to the household problem in (2.11) of the definition of a Recursive Competitive Equilibrium. A solution to this problem generates

$$
n = \widetilde{\eta}(k, \epsilon; \Gamma, \tau, \tau') \ , \ c = \widetilde{g}(k, \epsilon; \Gamma, \tau, \tau') \text{ and } k' = \widetilde{h}(k, \epsilon; \Gamma, \tau, \tau').
$$

It is instructive to understand how the savings choice varies across individual capital holdings and future tax rates for the evolution of the wealth distribution. Note that in Figure 5 higher future tax rates for a given k induce a lower level of savings.[12]

The primary reason why a solution to the politico-economic equilibrium is difficult to find is that the tax choice τ' and associated decision rule $\widetilde{h}$ induce a new sequence of distributions:

$$
\begin{aligned}
\Gamma' &= \widetilde{H}\left(\Gamma, \tau, \tau'\right) \\
\Gamma'' &= H\left(\widetilde{H}\left(\Gamma, \tau, \tau'\right), \tau'\right) \\
\Gamma''' &= H\left[H\left(\widetilde{H}\left(\Gamma, \tau, \tau'\right), \tau'\right), \Psi\left(\widetilde{H}\left(\Gamma, \tau, \tau'\right), \tau'\right)\right] \\
&\ldots
\end{aligned}
\tag{2.13}
$$

Because of this difficulty, Aiyagari and Peled [7] restricted off-the-equilibrium out-

[12]The figure plots $k' = \widetilde{h}(k, \epsilon; \Gamma, \tau, \tau')$ for $\epsilon_3 = 1$, all evaluated at the steady state distribution Γ associated with τ.

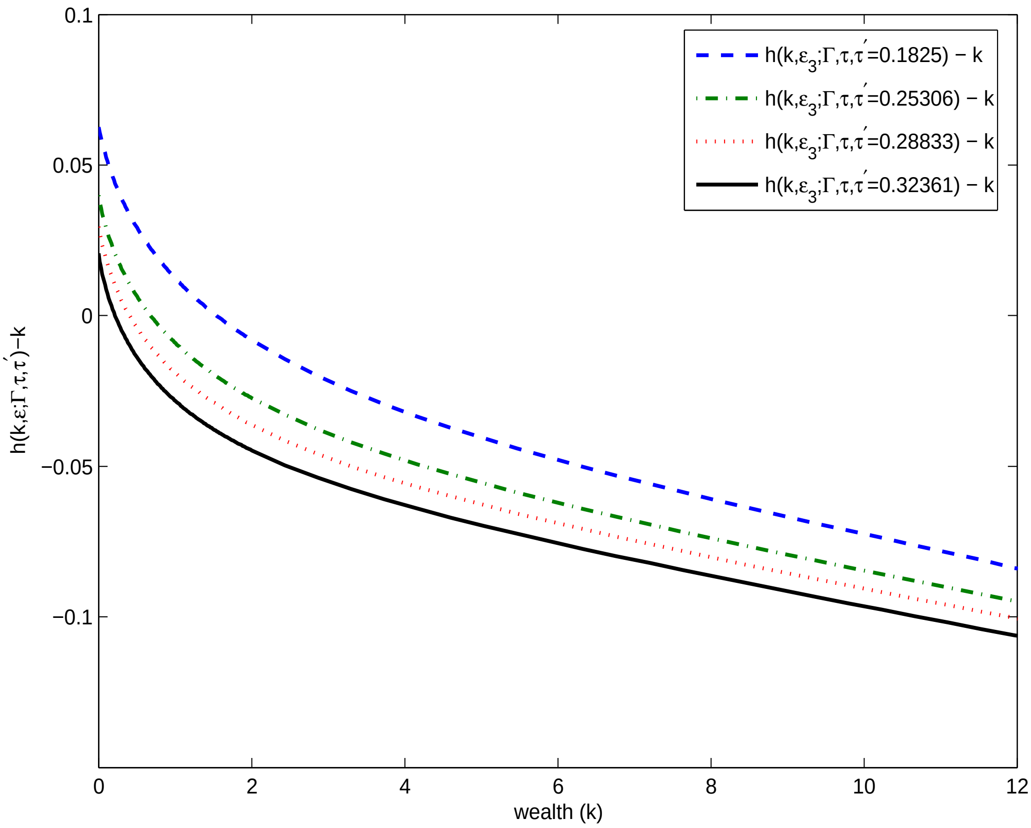


Figure 2.5: Decision rules over wealth for different levels of τ'.

comes to be steady states. Specifically, Aiyagari and Peled assume that $\Gamma'' = \Gamma^*(\tau^*)$ where Γ^* denotes the steady state distribution corresponding to tax choice τ'.

Next we define the solution concept.

Definition (PRCE) A **Politico-Economic Recursive Competitive Equilibrium** is:

(i) a set of functions $\{V, \eta, g, h, H, \Psi, r, w, T\}$ that satisfy the definition of a RCE;

(ii) a set of functions $\{\widetilde{V}, \widetilde{\eta}, \widetilde{g}, \widetilde{h}\}$ that solve (2.12), at prices which clear markets and the govt. budget constraint, and $\widetilde{H}$ satisfying

$$\Gamma'(k', \epsilon') = \int 1_{\{\widetilde{h}(k,\epsilon;\Gamma,\tau,\tau')=k'\}} \Pi(\epsilon'|\epsilon) d\Gamma(k, \epsilon)$$

with continuation values satsifying (i);

(iii) in individual state $(k, \epsilon)_i$, household $i's$ most preferred tax policy τ^i satisfies

$$\tau^i = \psi((k, \epsilon)_i, \Gamma, \tau) = \arg\max_{\tau'} \tilde{V}((k, \epsilon)_i, \Gamma, \tau, \tau'); \tag{2.14}$$

(iv) the policy outcome function $\tau^m = \Psi(\Gamma, \tau) = \psi((k, \epsilon)_m, \Gamma, \tau)$ satisfies

$$\begin{aligned} \int I_{\{(k,\epsilon):\tau^i \geq \tau^m\}} d\Gamma(k, \epsilon) &\geq \frac{1}{2} \\ \int I_{\{(k,\epsilon):\tau^i \leq \tau^m\}} d\Gamma(k, \epsilon) &\geq \frac{1}{2}. \end{aligned}$$

Condition (iv) effectively defines the median voter. That is, tax outcomes are determined by the voter whose most preferred tax rate is the median of the distribution of most preferred tax rates. To find the median voter, we sort the agents by their most preferred tax rates and then we integrate the distribution of most preferred tax rates over (k, ϵ) using $\Gamma(k, \epsilon)$.

For the existence of this type of politico economic equilibrium, preferences need to be single peaked.[13] Single-peakedness simply says that there is an alternative τ^i that represents a peak of satisfaction and, moreover, satisfaction increases as we approach this peak. We do not have a general proof of single peakedness; however, we check that in the calibrated economy we solve numerically, the indirect utility function satisfies this property *for every* $(k, \epsilon, \Gamma, \tau)$ including those off the equilibrium path.[14] Graphically we can see the importance of this condition from Figure 6. There we plot the indirect utility function $\widetilde{V}(k, \epsilon, \Gamma, \tau, \tau')$ over τ' for different households (k, ϵ) evaluated at $\tau = 0.365$ and the steady state distribution Γ associated with that τ. Generally, single-peakedness is used to establish that the median ranked preferred tax rate beats any other feasible tax rate in pairwise comparisons so that the median voter theorem applies.

[13]For household i in individual state $(k, \epsilon)_i$ and aggregate state Γ, τ, preferences of voter i are *single peaked* if the following condition holds: if $\tilde{\tau} \leq \hat{\tau} \leq \tau^i$ or if $\tilde{\tau} \geq \hat{\tau} \geq \tau^i$, then $\widetilde{V}((k, \epsilon)_i, \Gamma, \tau, \tilde{\tau}) \leq \widetilde{V}((k, \epsilon)_i, \Gamma, \tau, \hat{\tau})$.

[14]The papers by Azzimonti, et. al. [15] and Basetto and Benhabib [16] have proofs of single-peakedness in nonstochastic environments.

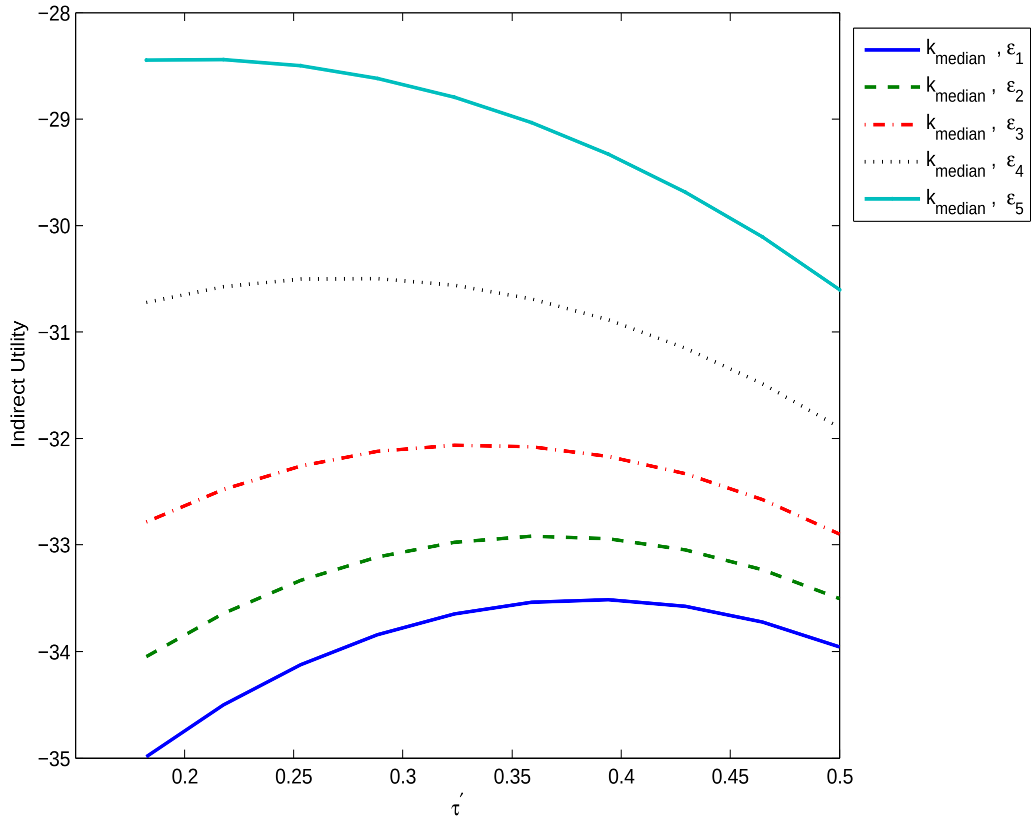


Figure 2.6: Single Peaked Preferences.

In our environment, the median voter identity is endogenous. In models without uncertainty or with complete markets, an agent with mean capital holdings would choose zero redistribution. However, in our model, even agents with the mean capital holding will vote for a positive tax rate for insurance reasons. A higher government transfer allows agents with low wealth to smooth consumption. There are also general equilibrium considerations. As τ increases, the household decision rule implies lower capital accumulation which results in a higher interest rate and lower wage rate. If the latter effect dominates, the distribution will compress.

Finally, we restrict attention to steady state equilibria of the above definition. Specifically,

Definition (SSPRCE). A **Steady State PRCE** is a PRCE which satisfies $\Gamma^* = H(\Gamma^*, \tau^*)$ and $\tau^* = \Psi(\Gamma^*, \tau^*)$.

2.3.4 Alternative Mechanisms

We compare our results with three alternative mechanisms. First, we analyze what would be the equilibrium tax rate if it is chosen by sequentially maximizing average welfare, i.e. the solution to a planner's problem with no commitment. We call it the utilitarian mechanism with no commitment. In this case and identical to the equilibrium considered in the previous section, no restrictions are imposed over the evolution of tax rates. Second, we consider median voter and the utilitarian mechanisms with commitment, that is where only a one time change in tax rates is allowed. More specifically, tax rates are restricted to be fixed after the first period.

Utilitarian Mechanism with no commitment

The planner sequentially chooses a future tax rate to maximize aggregate welfare. The definition of equilibrium is identical to that of a **PRCE** but where the condition that defines the equilibrium tax function, condition (iv), is replaced by:

$$\Psi^{un}(\Gamma, \tau) = \arg\max_{\tau'} \int \widetilde{V}(k, \epsilon, \Gamma, \tau, \tau')d\Gamma(k, \epsilon).$$

with all continuation values evaluated according to the equilibrium function (e.g. $\tau'' = \Psi^{un}(\Gamma', \tau')$). As before changes in tax rates affect the evolution of the wealth distribution and viceversa.

Mechanisms with commitment

We consider two other tax choice mechanisms with commitment.[15] The first is a simple restriction on the PRCE defined above. In particular, the median voter chooses a future permanent tax rate. It is as if the government can commit to the tax rate. Specifically, the only constraint on problem PRCE is that all continuation values are evaluated according to the "identity" function (that is, $\tau_{t+n+1} = \Psi(\Gamma_{t+n}, \tau_{t+n}) = \tau_{t+n}$, for all Γ_{t+n} and τ_{t+n}, $n = 1, 2, ...$ with $\tau_{t+1} = \Psi^O(\Gamma, \tau) = \arg\max_{\tau'} \widetilde{V}((k, \epsilon)_m, \Gamma, \tau, \tau')$. Note that in this case we restrict only the evolution of tax rates. The evolution of the joint distribution Γ is given by the equilibrium function $H(\Gamma, \tau)$. It is still necessary to compute the entire transition of prices for each possible tax change. We call this case the *one-time median voter tax choice.*

Even for the one-time voting case, there is a nontrivial transition path for the wealth distribution similar to (2.13). Specifically, we have

$$
\begin{aligned}
\Gamma' &= \widetilde{H}\left(\Gamma, \tau, \tau'\right) \\
\Gamma'' &= H\left(\widetilde{H}\left(\Gamma, \tau, \tau'\right), \tau'\right) \\
\Gamma''' &= H\left[H\left(\widetilde{H}\left(\Gamma, \tau, \tau'\right), \tau'\right), \tau'\right] \\
&\ldots
\end{aligned}
$$

Figure 7 displays the transition paths of aggregate capital for different one-time changes in tax rates.[16] The starting point is the aggregate capital corresponding to the invariant distribution $\Gamma^*(\tau^*)$ with constant taxes for the initial SS calibration. Higher future tax rate choices $\widehat{\tau} > \tau^*$ imply aggregate capital paths that are mono-

[15] Besides providing an interesting theoretical contrast to the sequential problem, from a computational standpoint the one-time problem is much quicker and can serve as a useful starting point for the sequential case.

[16] This corresponds to point $(3.b)$ in the computational algorithm and the discussion immediately following for one-time tax changes.

tonically decreasing. Higher future tax rates generate decreases in individual savings that are reflected in these paths to the new invariant distribution $\widehat{\Gamma}(\widehat{\tau})$ associated with $\widehat{\tau}$. The effects of the tax change disappear slowly (about 50 model periods or years).

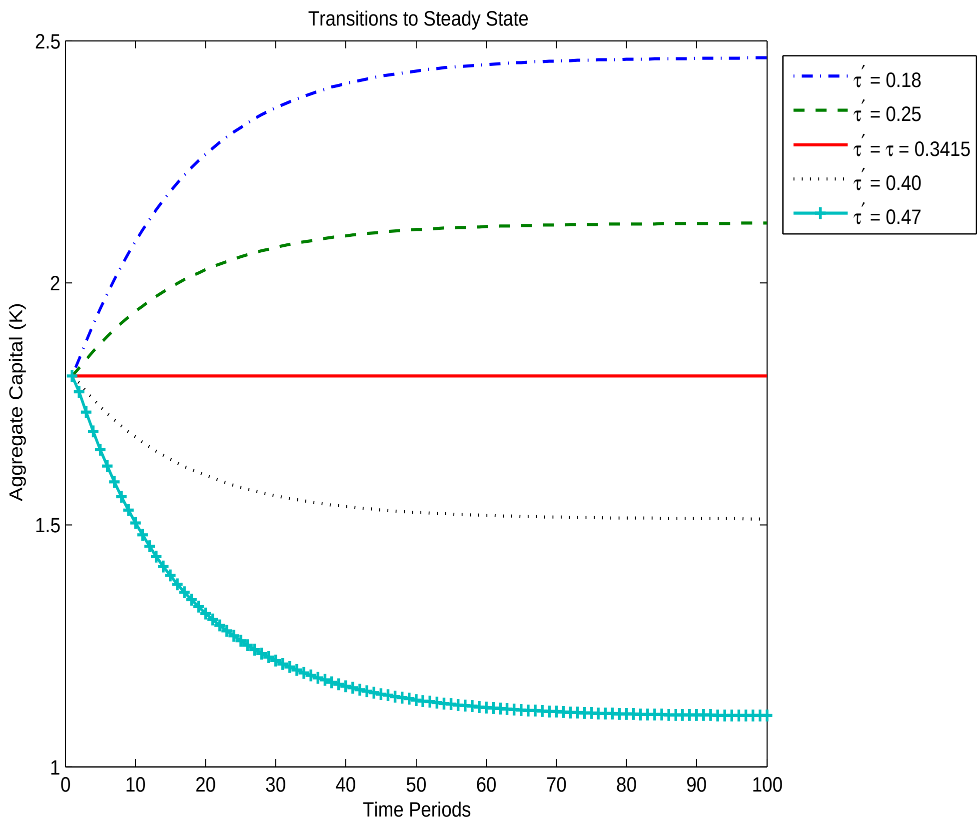


Figure 2.7: Transitions at initial steady state τ

To contrast to this mechanism, we consider a *one-time utilitarian tax choice*. In this case, the planner chooses a future constant tax rate to maximize aggregate welfare:

$$\Psi^{uc}(\Gamma, \tau) = \arg\max_{\tau'} \int \widetilde{V}(k, \epsilon, \Gamma, \tau, \tau')d\Gamma(k, \epsilon).$$

with all continuation values evaluated according to the "identity" function (e.g.

Table 2.1: Preferences and Technology Parameters.

Parameter		Value
Discount Factor	β	0.96
Preferences	γ	1
	ν	0.3
	χ	75
Capital Share	α	0.36
Depreciation Rate	δ	0.06

$\tau'' = \Psi(\Gamma', \tau') = \tau' \quad \forall \Gamma',\ \tau')$.

2.4 Calibration

We calibrate the model to the U.S. economy. We can group the parameters in two different sets: (i) preferences and technology $\{\beta, \gamma, \nu, \chi, \alpha, \delta\}$; and (ii) the wage generating process $\{E, \Pi\}$. The first group is set to standard values in the RBC literature. The second set of parameters is obtained by directly computing mobility (i.e. transition) matrices for hourly wage rates in the PSID data from 1978 to 1979 (corresponding to the low inequality regime) and from 1995 to 1996 (corresponding to the high inequality regime).

2.4.1 Preference and Technology parameters

Some of the preference and technology parameters (β, γ, α, and δ) are set to standard values for the U.S. economy when using a neoclassical growth of model. The intertemporal Frisch elasticity ν is estimated to be between 0.1 and 0.45 for prime age males by McCurdy (1981). We take ν to be 0.3. The parameter χ is set so that aggregate effective labor supply is equal to 0.3 in 1979 as in Heathcote [61]. The value of the parameters are displayed in table (2.1). The time period chosen for the model is four years.

Table 2.2: Transition Matrix for 1978-1979.

	1 (6.84849)	2 (10.93091)	3 (14.88623)	4 (19.51769)	5 (30.70179)
1 (6.728081)	0.672284357	0.23663245	0.058679332	0.021755784	0.010648077
2 (10.97251)	0.230851716	0.491191789	0.213694853	0.043428309	0.020833333
3 (14.84763)	0.058963349	0.217834688	0.524306088	0.152507284	0.046388591
4 (19.40453)	0.032651184	0.041465471	0.15053269	0.575764544	0.199586112
5 (31.55262)	0.007289748	0.010282382	0.053330264	0.207105586	0.72199202

2.4.2 Wage process

We set the number of elements in E to five since much of the effective tax rate data we consider is in terms of quintiles (so $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\}$ where ε_i refers to average wage rate of individuals in wage quintile i). We use the PSID data to obtain the annual mobility matrices (transition probabilities) from 1978 to 1979 and from 1995 to 1996. We restrict our sample to household heads who are between ages 20 and 59, whose annual hours of work is between 520 hours and 5096, and who earn at least half of the minimum wage, and who are in the sample for both years for the years that we calculate the transition matrices. Moreover, we use population weights when we compute our transition matrices. Given this we obtain the following mobility matrices

Since average wages in each year are not the same we take the average of the two consecutive years as our ε_i. For example, ε_1 for the first transition matrix is (6.84849+6.728081)/2.

Our selection criteria implies that the variance of log wages increases from 0.28 before 1979 to 0.37 in 1996 while the median to mean ratio declines from 0.9 to 0.79. For comparison, Heathecote et. al. [62] report that the variance of log wages

Table 2.3: Transition Matrix for 1995-1996.

	1 (6.643506)	2 (10.55364)	3 (14.33294)	4 (19.87673)	5 (38.81428)
1 (6.343664)	0.714443533	0.199974569	0.03948081	0.021702287	0.024398801
2 (10.27176)	0.181288293	0.514990414	0.233577389	0.048408029	0.021735875
3 (13.78551)	0.0491733	0.266339048	0.497278842	0.164298927	0.022909882
4 (19.16724)	0.039455357	0.012745583	0.19145693	0.61755159	0.13879054
5 (38.0832)	0.017569988	0.004346983	0.03262434	0.146594941	0.798863747

increased from 0.28 to 0.39 and median to mean ratio declined from 0.9 to 0.76. To get a sense of the approximation error associated with our transition matrices, we note that the implied ratio of median to mean wages are 0.885 and 0.800 in 1979 and 1996 respectively and the implied variance of log wages are 0.265 and 0.34 in 1979 and 1996 respectively. Since we are grouping individuals in wage brackets, it is expected that the level and changes in these inequality measures implied by these transition matrices are smaller. However, the approximation error is still quite small.

2.4.3 Data Description for Wage Process

Here we describe the data and steps we use to construct labor earnings tranisition matrices: one for transitions from 1978 to 1979 and another from 1995 to 1996. There are five states in each year. Household heads in the first state have the lowest real hourly wages and those in the fifth state have the highest real hourly wages. The publicly available data set we use is the Panel Study of Income Dynamics (PSID). To make it more convenient for anyone who wants to replicate our results, the specific PSID variable names are included. The weight variables we use are the 1979 weight

(V7451) and the 1996 weight (ER12084). [17]

1. The nominal hourly wages of household heads are calculated by taking the nominal annual labor earnings divided by annual work hours. The nominal annual labor earnings of household heads (V6767, V7413) and annual work hours of household heads (V6336, V6934) are readily given for 1978 and 1979. To obtain these two measures for the years 1995 and 1996, the following extra steps need to be taken:

   - Add up labor earnings from various sources to obtain nominal annual labor earnings of household heads: wages (ER8256, ER11150), bonuses (ER8263, ER11157), commissions (ER8266, ER11160), overtime (ER8264, ER11158), professional practice or trade (ER8270, ER11163), tips (ER8265, ER11159), market gardening (ER8286, ER11179), roomers or boarders (ER8302, ER11195), and extra jobs (ER8318, ER11211).
   - If household heads are currently working in the year when surveyed, the annual work hours in the previous year are the number of weeks worked per year for main jobs (ER7317, ER10231) times the number of hours worked per week for main jobs (ER7320, ER10232) plus the number of weeks worked per year for extra jobs (ER7327, ER10239) times the number of hours worked per week for extra jobs (ER7328, ER10240). If the household heads are not currently working, all the above variables are coded zero.
   - If the household heads are not currently working in the year when surveyed, the annual work hours in the previous year can be obtained similarly by taking the the number of weeks worked per year for main jobs

[17] We note that the surveys asked households the history of previous year instead of the year in which the surveys were conducted.

(ER7562, ER10470) times the number of hours worked per week for main jobs (ER7565, ER10471) plus the number of weeks worked per year for extra jobs (ER7572, ER10478) times the number of hours worked per week for extra jobs (ER7573, ER10479). If the household heads are currently working, all the above variables are coded zero.

- The annual work hours not conditional on current working status for a household head is the larger number of the two calculations above because by construction the smaller number will be zero. Note that hours worked per week should be treated as missing values if coded as 998 or 999. The weeks worked per year should be treated as missing values if coded as 98 or 99.

2. Next, nominal hourly wages are adjusted to be in 1992 dollars. The nominal hourly wages are either deflated or inflated using the Consumer Price Index - All Urban Consumer (CPI-U) from U.S. Department of Labor, Bureau of Labor Statistics.

3. The sample selection criteria are: (1) the age of the household head (V6462, V7067, ER7006, and ER10009) is between 20 and 59, (2) the household head works for no less than 520 hours and no more than 5096 hours annually, (3) the nominal hourly wage of the household head is greater than half of the federal minimum wage rate (available from U.S. Department of Labor, Employment Standards Administration Wage and Hour Division), (4) the nominal annual labor earnings of the household head are less than $99999 for the years 1978 and 1979 and less than $999996 for the years 1995 and 1996, (5) the household satisfies all of the above criteria in both years 1978 and 1979 and/or in both years 1995 and 1996.

4. For the households who satisfy all the sample selection criteria, the state they

are in can now be assigned for each year. If the real hourly wage earned by the household head is higher than at least $20*(i\text{-}1)\%$ but no more than $20*i\%$ of the samples, this household is in the ith state, $i \in \{1, 2, 3, 4, 5\}$.

5. Finally, we are ready to build the 5-by-5 transition matrix Π for the year 1978-79. Each row or column represents a state. The frequencies of each cell in the matrix are counted. If a household is in the ith state in 1978 and in the jth state in 1979, then he will be counted one in cell $\Pi(i, j)$. For each cell, we divide the counts by total counts of the entire row the cell is in to get the sample proportions conditional on the state of year 1978. The transition matrix is completed when the frequencies are replaced by the conditional sample proportions for all cells. Repeat to get the transition matrix for the year 1995-96.

2.4.4 Government Spending

We next calibrate certain parameters of the left hand side of the government budget constraint (2.4). Since our model abstracts from retirement and the reasons for federal government spending like defense, we include social security transfers as part of government spending (i.e. it is a resource lost on agents not in the model). Using this categorization for 1979, 5.2% of GDP was associated with social security and 9.1% of GDP was associated with government purchases yielding $G_{1979} = 9.1+5.2 = 14.3$. In 1996, 7% of GDP was associated with social security and 5.3% of GDP was associated with government purchases yielding $G_{1996} = 5.3 + 7 = 12.3$.[18]

[18]The data comes from Table 15.5 (Total Government Expenditures by Major Category of Expenditure as Percentages of GDP: 1948–2006) on the U.S. Government Printing Office web page under Budget of the United States Government: Historical Tables Fiscal Year 2008. The link to the table is http://www.gpoaccess.gov/usbudget/fy08/sheets/hist15z5.xls.

2.4.5 Data Description for Federal Effective Tax Rates

The effective tax rate measures the percentage of household income going to the federal government from taxes. The income measure is comprehensive household income, which comprises pretax cash income plus income from other sources. Pretax cash income is the sum of wages, salaries, self-employment income, rents, taxable and nontaxable interest, dividends, realized capital gains, cash transfer payments, and retirement benefits plus taxes paid by businesses (corporate income taxes; the employer's share of Social Security, Medicare, and federal unemployment insurance payroll taxes); and employees' contributions to 401(k) retirement plans. Other sources of income include all in-kind benefits (Medicare, Medicaid, employer-paid health insurance premiums, food stamps, school lunches and breakfasts, housing assistance, and energy assistance). Households with negative income are excluded from the lowest income category but are included in totals.

We calculate federal effective taxes for nonelderly households. To do that we use Table 2C and 4C from from "Effective Federal Tax Rates for All Households" from http://www.cbo.gov/showdoc.cfm?index=7000&type=1. Table 2C reports the number of households, average pretax income, and average after-tax income for each income quintile for households with children, i.e. a household that has at least one member under age 18. Table 4C reports the same statistics for nonelderly childless households, i.e. a household headed by a person under age 65 and with no member under age 18. The two groups make up all nonelderly households plus elderly housholds with children under 18. The CBO does not provide data that would allow us to exlude elderly housholds with children under 18. However, the size of the elderly housholds with children under 18 group is rather small and is unlikely to affect our calculations. Therefore, we combine the two groups to represent nonelderly households. Using number of households, average pre-tax income, and average after-tax income for each quintile in each group, we calculate total pre-tax income and total tax liability (pre-tax income minus after-tax income) for each quintile in each group and use these to compute the total pre-tax income and total tax liability of the combined group. Then we divide total tax liability of each quintile by the total pre-tax income of that quintile in the combined group to get the effective taxes by quintile for the combined group.

2.5 Quantitative Exercise

To assess the quantitative significance of the change in inequality for changes in effective taxes, we feed the transition matrix for wage rates from 1978 to 1979 into the model to deliver a steady state effective tax rate in the initial regime. Then we feed the transition matrix for wage rates from 1995 to 1996 into the model to deliver a steady state effective tax rate in the final regime.

After solving the saving decision problem of the household we can solve problem (2.14) in the definition of PRCE to obtain the tax rate that maximizes each agent's utility. In Figure 2.8 we observe the most preferred tax rates as a function of k for different levels of ϵ. The feasible set of tax rates is restricted to the interval $[0, 1]$. For a fixed level of wealth k, the function $\tau' = \psi(k, \epsilon, K, \tau)$ is decreasing in ϵ. That is, for a given level of assets, an agent with the lowest productivity ϵ_1 will vote for a higher tax rate than an agent with higher productivity levels ϵ_2 to ϵ_5. This implies that the fraction of households in each productivity level is critical for the determination of the optimal tax rate.

Clearly if two households have equal productivity levels at the time of the tax reform, but different levels of wealth k, the wealthier household has more to lose from an increase in tax rates. This effect is seen as a movement along $\tau' = \psi(k, \epsilon, K, \tau)$ for a given ϵ in Figure 2.8. The figure shows that the optimal tax rate is decreasing in the level of wealth for a given level of labor productivity. Wealthier agents receive a large portion of their income from the return on capital and therefore changing the tax rate affects the expected net return. In general, this effect offsets the effect of the increase in the government transfers mentioned above.

Finally, Figure 2.8 shows that it is possible for households with two different (k, ϵ) to choose the same tax rate τ' (this is seen as a horizontal slice). For instance, it is evident that a household with $(1.2, \epsilon_3)$, one with $(1.9, \epsilon_2)$ and one with $(2.2, \epsilon_1)$ choose the same tax rate $\tau' = 0.365$.

We can summarize the tax choice of a typical agent as follows:

1. For a given (k, Γ, τ), $\psi(k, \epsilon, \Gamma, \tau)$ is decreasing in ε; that is, a household with a lower wages will choose a higher τ'.

2. For a given $(\epsilon_i, \Gamma, \tau)$, $\psi(k, \epsilon, \Gamma, \tau)$ is decreasing in k; that is, a household with a lower wealth will choose a higher τ'.

3. For a given (Γ, τ), there may be households with different wealth and wages who choose the same τ'.

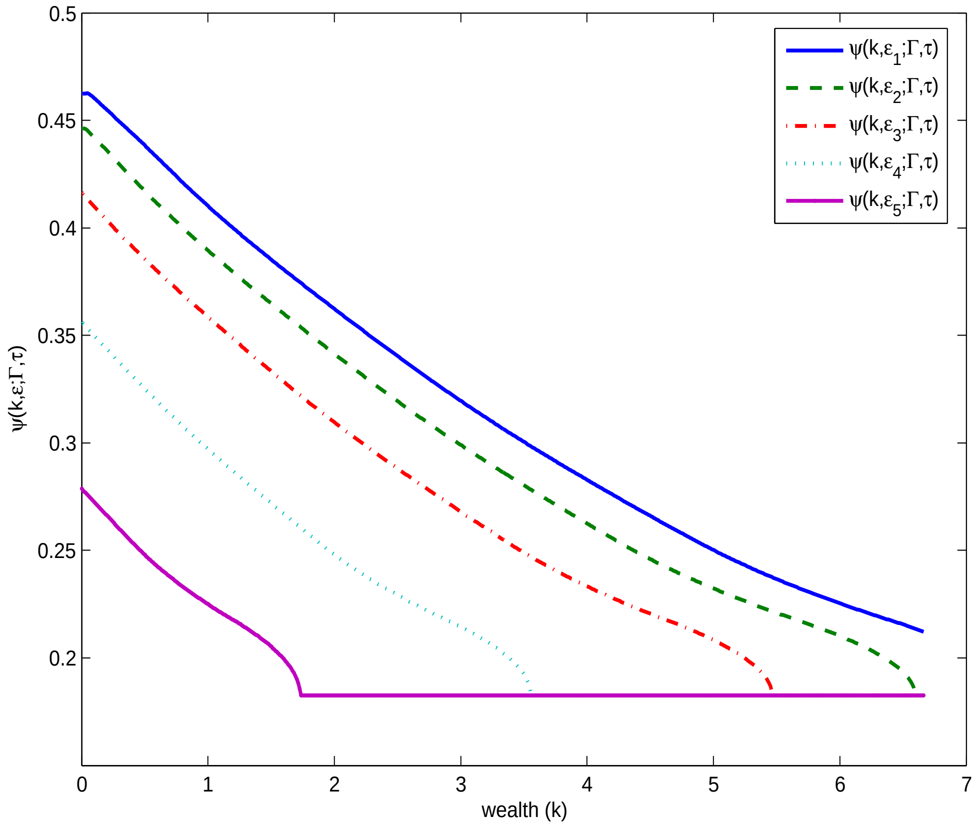


Figure 2.8: Most Preferred Tax Rate.

To take the theoretical marginal tax rate τ to the data, we use the CBO's definition of effective tax rates, which we denote e. It is defined to be the amount of tax liability divided by pre-tax income including transfers. In the data, the tax liability is reported net of earned income tax credit and this is not included in the transfer measure. That is, from the total transfer T some fraction $\phi \in [0, 1]$ is computed as a credit in income taxes and the rest $(1 - \phi)$ is finally distributed as a pure transfer. Thus, for accounting reasons, let $\Upsilon = \phi T$ denote the earned income credit and $T^f = (1 - \phi)T$ denote pure transfers. In the context of our model, the effective income tax rate is given by:

$$e = \frac{\tau \int (rk + w\epsilon) d\Gamma(k, \epsilon) - \Upsilon}{\int (rk + w\epsilon) d\Gamma(k, \epsilon) + T^f}. \tag{2.15}$$

The parameter ϕ is calibrated as follows. At the given parameters, $\{\beta, \sigma, \alpha, \delta, E, \Pi\}$, we obtain the equilibrium marginal tax rate τ. We then choose ϕ to match the ratio of Total Earned Income Tax Credit to GDP ($\phi T/Y$) in 1996. The IRS reports that the Total Earned Income Tax Credit is \$22.1 billion. Nominal GDP from NIPA tables is \$7816.9 billion. To make a fair comparison between the different mechanisms and because each mechanism generates a different marginal tax rate (and transfers), ϕ varies from one mechanism to the other. Specifically, we find $\phi = 0.0109$ for the sequential mechanism and $\phi = 0.0117$ for the utilitarian mechanism.

Equation 2.15 implies that the effective tax rate increases with income. We illustrate this simple progressive tax system in Figure 2.9. The slope of the red dotted line gives the effective tax rate. As can be seen from this figure the effective tax rate increases as income increases even if the marginal tax rate is independent of income (as in the case of our model).

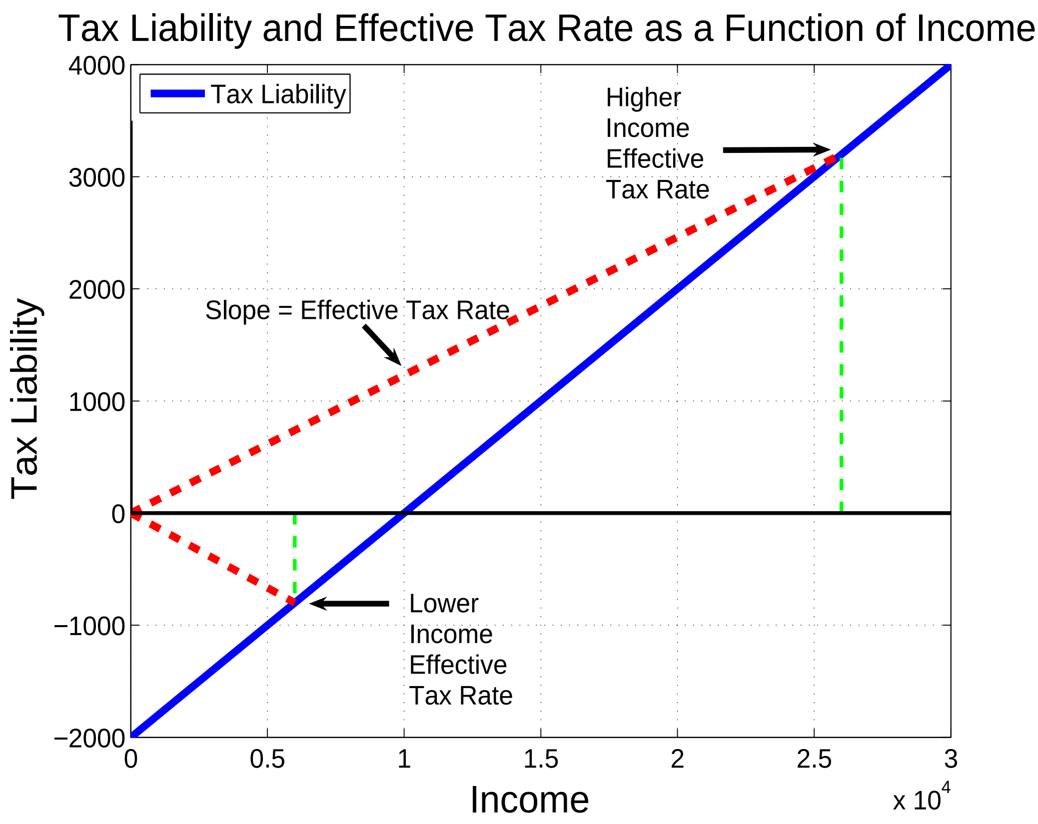


Figure 2.9: Progressive Tax System

Table (2.4) presents the changes in effective income tax rates by income quintile when normalized by the middle quintile, the analogue of our Figure 3. The model is not capable of matching the big changes that we observe in the data for the lowest quintiles and tends to overpredict changes in the highest quintile.

As suggested in Section 2, rising inequality by itself could potentially generate a rise in effective tax rates without any change in the marginal tax rate τ through the effect of changes in labor income working through a progressive tax system. While the estimates by Harris, et. al. [59] we present in Figure 2.4 suggest that the changes in effective taxes due only to changes in income are rather small, we can run a counterfactual to decompose how much of the change in effective tax

Table 2.4: Effective Income Tax Rate by Quintile (Normalized by Middle Quintile)

Effective Tax Rates	Quintiles	Normalized		%Δ
		1979	1996	
	Q1 (lowest)	0.4997	0.320	-35.91
	Q2	0.8233	0.776	-5.76
Data	Q3 (middle)	1	1	0
	Q4	1.1165	1.117	0.05
	Q5 (highest)	1.3839	1.454	5.08
	Q1 (lowest)	0.8068	0.7259	-10.02
One-time	Q2	0.9355	0.9004	-3.75
Median Voter	Q3 (middle)	1.0000	1.0000	0
	Q4	1.0479	1.0824	3.29
	Q5 (highest)	1.1002	1.1800	7.26
		Initial SS	Final SS	
	Q1 (lowest)	0.7943	0.7247	-8.77
One-time	Q2	0.9308	0.8997	-3.33
Utilitarian	Q3 (middle)	1.0000	1.0000	0
	Q4	1.0518	1.0830	6.58
	Q5 (highest)	1.1083	1.1812	6.58
	Q1 (lowest)	0.7898	0.676	-14.340
Seq.	Q2	0.9287	0.878	-5.513
Median Voter	Q3 (middle)	1.0000	1.0000	0
	Q4	1.0537	1.107	5.030
	Q5 (highest)	1.1125	1.234	10.876
	Q1 (lowest)	0.760	0.688	-9.504
Seq.	Q2	0.917	0.883	-3.761
Utilitarian	Q3 (middle)	1.000	1.000	0
	Q4	1.065	1.102	3.398
	Q5 (highest)	1.136	1.222	7.535

Table 2.5: Fraction of changes in normalized effective tax rates due only to changes in wages

	Sequential	Utilitarian
Q1	46%	74%
Q2	47%	74%
Q3		
Q4	46%	47%
Q5	48%	77%

rates in 1996 is attributable to changes solely in the wage process (something that is virtually impossible to do in the data) using our model. Specifically, we impose the sequential equilibrium marginal tax rate chosen by the median voter τ in the low inequality (1979) regime into a competitive equilibrium from the high inequality (1996) regime.[19] This gives us a counterfactual set of effective tax rates for the 1996 regime that are attributable only to changes in the wage process. We then use these tax rates to obtain effective tax rates across quintiles and normalize them as we did earlier. Then we calculate the percentage changes in these counterfactual normalized tax rates. This gives us the percentage change in normalized tax rates due to the change in the wage process. Then we compute the ratio of the percentage change in counterfactual normalized effective tax rates to percentage change in actual normalized effective tax rates to obtain the numbers in Table 2.5. As evident in the table, the sequential mechanism attributes less change in effective tax rates due to wage changes than the utilitarian mechanism. Thus the results from the sequential mechanism are closer to the findings of Harris, et. al. [59] than the results from the utilitarian mechanism.

There is one key observational difference between our work and the previous political economy models mentioned in the introduction. Models that do not

[19] In other words, we simply solve an Aiyagari [6] economy calibrated to 1996 with τ set at the level implied by our SEQ for 1979.

incorporate idiosyncratic uncertainty generate a direct relation between wealth and preferred tax rates; that is, households with more wealth than the median level always vote for lower taxes and the opposite is true for households with lower than median wealth. On the other hand, as evident in Figure 2.8, households with different levels of wealth k may vote for the same τ'. Figure 2.10 shows how agents vote in our model for different levels of wealth relative to the median voter. The figure is constructed as follows. After solving for the optimal tax rate we know the capital holdings of the median voter k_m (as well as his earnings). Then households are sorted based on their level of capital relative to k_m to form two groups: those with $k \geq k_m$ and those with $k \leq k_m$. Finally in each of these two groups, agents are separated between those who prefer a higher tax rate and those who prefer a lower tax rate than the median voter. The figure reports the normalized (relative to the number of households in the $k \geq k_m$ group and the $k \leq k_m$ groups) fraction who prefer higher or lower tax rates.

The panel on the left of Figure 2.10 shows the portion of agents with **lower** wealth k than the median voter. From this group only 62% vote for higher taxes (either those with lower earnings or those with extremely low capital and higher earnings) while 38% vote for lower taxes than the median voter (those with higher earnings). The panel on the right shows the portion of agents with **higher** capital than the median voter. In this case, only 9% vote for higher taxes (those with lower earnings level) while 91% vote for lower taxes than the median voter (either those with higher earnings or those with extremely high capital and lower earnings).

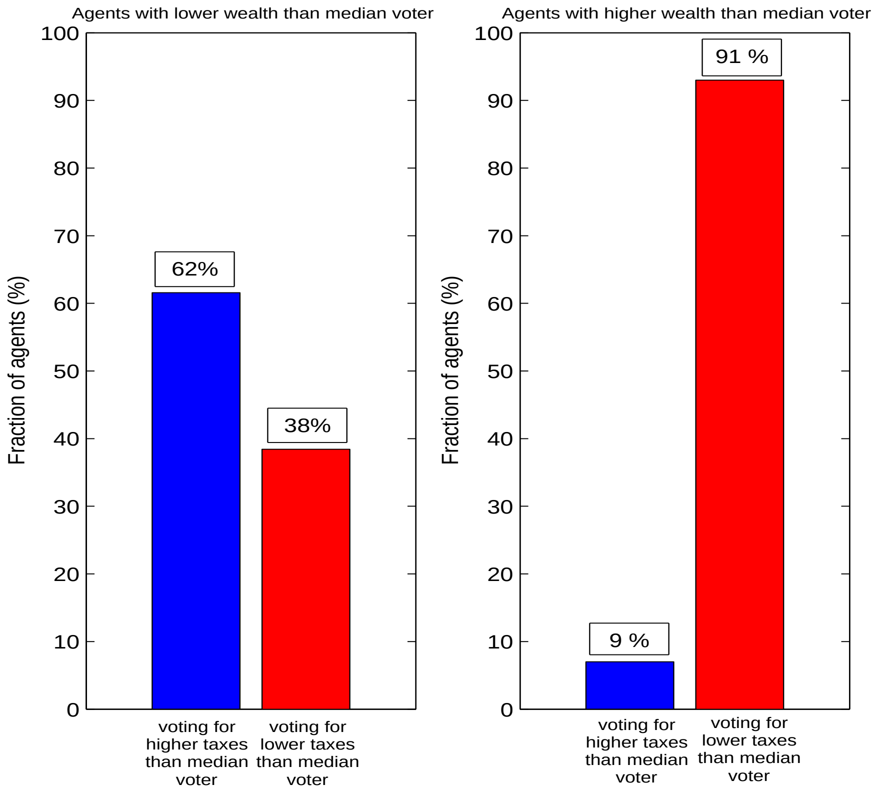


Figure 2.10: Distribution of Wealth and Tax Choices.

2.6 Welfare analysis

In this section we assess the welfare gains of endogenizing policy choices in response to the change in the underlying earnings process. In particular, we ask how much agents are willing to pay (in consumption equivalent terms) to use the equilibrium tax function chosen by the median voter in response to the high earnings inequality environment rather than sticking with the previous tax rates chosen in the low earnings inequality regime. In some ways, it measures the value of the insurance provided by a flexible tax policy.

More formally, the welfare gain (under mechanism m) for a household in state (k, ϵ) is defined as the constant percentage increased in consumption $\lambda^m(k, \epsilon)$, after the increase in wage inequality but under the constant tax rate chosen in 1979 (τ_{79}^m) that allows the household to achieve the same expected utility as when taxes are adjusted according to the equilibrium functions of mechanism m in 1996 ($H^m(\Gamma, \tau; E_{96})$ and $\Psi^m(\Gamma, \tau; E_{96})$) - i.e. those derived from the equilibrium with high wage inequality. Mechanism m can be the one-time utilitarian, one-time median voter, sequential utilitarian or sequential median voter.

Let $\Gamma_{79}^m(k, \epsilon; E_{96})$ be the steady state distribution when the tax rate is constant at τ_{79}^m and the calibration of the wage process correspond to that of year 1996. Furthermore, denote by $c_t(\Gamma_{79}^m(k, \epsilon; E_{96}), \tau_{79}^m)$ and $n_t(\Gamma_{79}^m(k, \epsilon; E_{96}), \tau_{79}^m)$ the optimal consumption and labor choice in period t at the steady state distribution $\Gamma_{79}^m(k, \epsilon; E_{96})$, that is when the tax rate is kept constant at the equilibrium tax rate of mechanism m in year 1979 (τ_{79}^m) and the earnings process is that of 1996 (E_{96}).

Let $V(k, \epsilon; \Gamma, \tau, H^m(\Gamma, \tau; E_{96}), \Psi^m(\Gamma, \tau; E_{96}))$ denote the value of an agent at state (k, ϵ) when the equilibrium law of motions for Γ and τ correspond to the equilibrium law of motion of mechanism m under the 1996 calibration as in equation (11). Then, the welfare gain $\lambda^m(k, \epsilon)$ solves the following equation:

$$\begin{aligned} V(k, \epsilon; \Gamma_{79}^m(k, \epsilon; E_{96}), \tau_{79}^m, H^m(\Gamma, \tau; E_{96}), \Psi^m(\Gamma, \tau; E_{96})) = \\ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t(\Gamma_{79}^m(k, \epsilon; E_{96}), \tau_{79}^m)(1 + \lambda^m(k, \epsilon)), n_t(\Gamma_{79}^m(k, \epsilon; E_{96}), \tau_{79}^m)), \end{aligned} \tag{2.16}$$

i.e. the welfare gain $\lambda^m(k, \epsilon)$ is the constant percentage increase in consumption that allows the household to obtain the same utility of switching to an economy where the law of motion corresponds to those of 1996, $H^m(\Gamma, \tau, E_{96})$ and $\Psi^m(\Gamma, \tau, E_{96})$, starting from the steady state distribution $\Gamma_{79}^m(k, \epsilon; E_{96})$ and tax rate τ_{79}^m.

The average welfare change measured in consumption equivalents is given by:

$$W^m = \int_{K \times E} \lambda^m(k, \epsilon) d\Gamma^m_{79}(dk, d\epsilon; E_{96}). \tag{2.17}$$

Using equations (2.16) and (2.17) we calculate expected welfare gains for households with various initial combinations of wealth and productivity. These numbers are computed by first creating a large artificial population, each member of which starts out with the initial wealth and productivity level of interest. The economy is then simulated forward (using the appropriate equilibrium sequence for prices and taxes) under both scenarios for tax policies.[20] Table 2.6 displays the average welfare gain for each mechanism m.

Table 2.6: Welfare Gains

Mechanism (m)	τ^m_{79}	τ^m_{96}	W^m (%)
One-time Median Voter	0.33	0.37	0.19
One-time Utilitarian	0.35	0.38	0.10
Seq. Median Voter	0.37	0.46	-0.40
Seq. Utilitarian	0.41	0.44	-0.21

The first thing to notice is that average welfare gains are very different across mechanisms. From Table 2.6 we observe that while average welfare increases for mechanisms with commitment (One-time), the opposite result is obtained when we analyze mechanisms without commitment (Sequential). For example the average expected welfare gain is equivalent to a permanent increase of 0.19% in the case of the One-time Median Voter equilibrium versus an average welfare loss of 0.40% in the case of the Sequential Median Voter.

[20]To obtain the consumption equivalent, we use 6000 initial combinations for (k, ϵ) (1200 initial values for k and the 5 values of ϵ). For each combination of (k, ϵ), we simulate the economy forward for 1000 periods and repeat it 100 times. The consumption equivalent $\lambda^m(k, \epsilon)$ is the average over the 100 repetitions.

To understand these results and because policy changes redistribute income and consumption across households differently, in Figure 2.11 we plot the welfare gain over wealth for different wage levels for the Sequential Median Voter mechanism.[21] This figure shows that welfare gains are decreasing in wealth, i.e at a given wage level, as wealth increases the welfare gain decreases. The increase in inequality is associated with an increase in τ. Keeping wages constant, as the fraction of income coming from capital gains increases, agents suffer more from a tax increase, so the welfare gain decreases. Furthermore, we observe that welfare gains are decreasing in wage levels, i.e. at a given wealth level, as the wage increases the welfare gain decreases. The intuition here is similar, keeping wealth constant, as the fraction of taxable income (in this case coming from labor) increases the welfare measure decreases.

[21] A similar figure can be obtained for the other mechanisms.

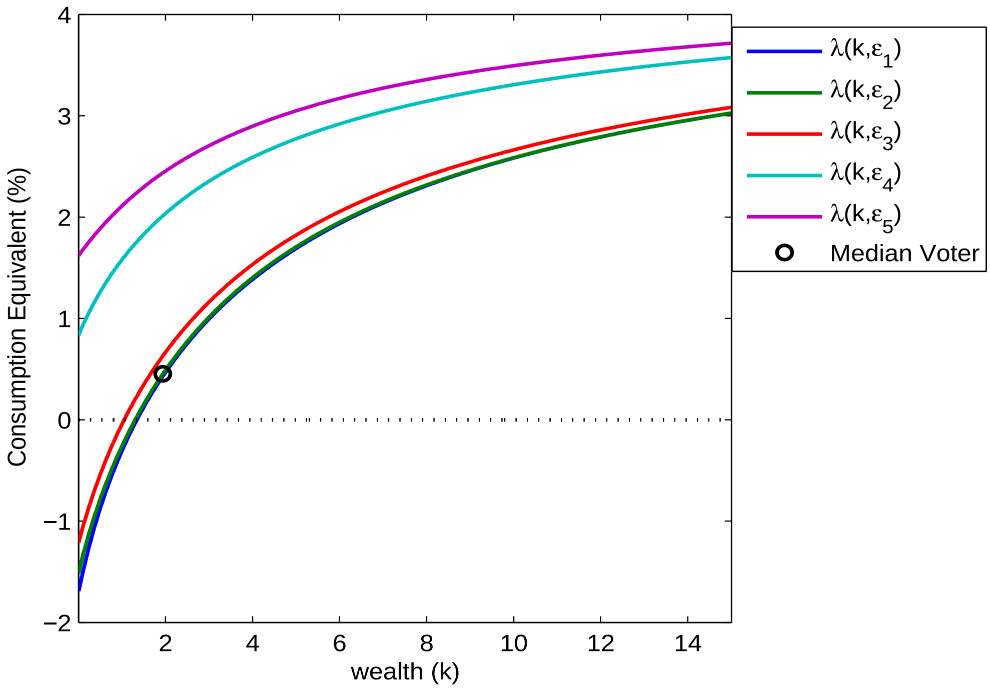


Figure 2.11: Welfare Gains $\lambda(k, \epsilon)$

Given all the heterogeneity in the model, it is important to note that the identity of the median voter in this economy corresponds to an agent with $(k = 1.2, \epsilon_3)$ and $\lambda(k = 1.2, \epsilon_3) > 0$; that is the median voter is in favor of the reform. Because the fraction of agents is not constant across wealth and wage levels, in Figure 2.12 we provide a histogram over welfare gains. We observe that around 60% of the population prefer the status quo, i.e. they obtain a negative welfare gain. Moreover, the range for losses is bigger than the range of gains. In particular, we note that expected losses are larger in absolute value than the average welfare gain. In Figure 2.13 we decompose this histogram by different wage levels. We observe that most of the winners correspond to agents with ϵ_1 and most of the losers correspond to agents with ϵ_5.

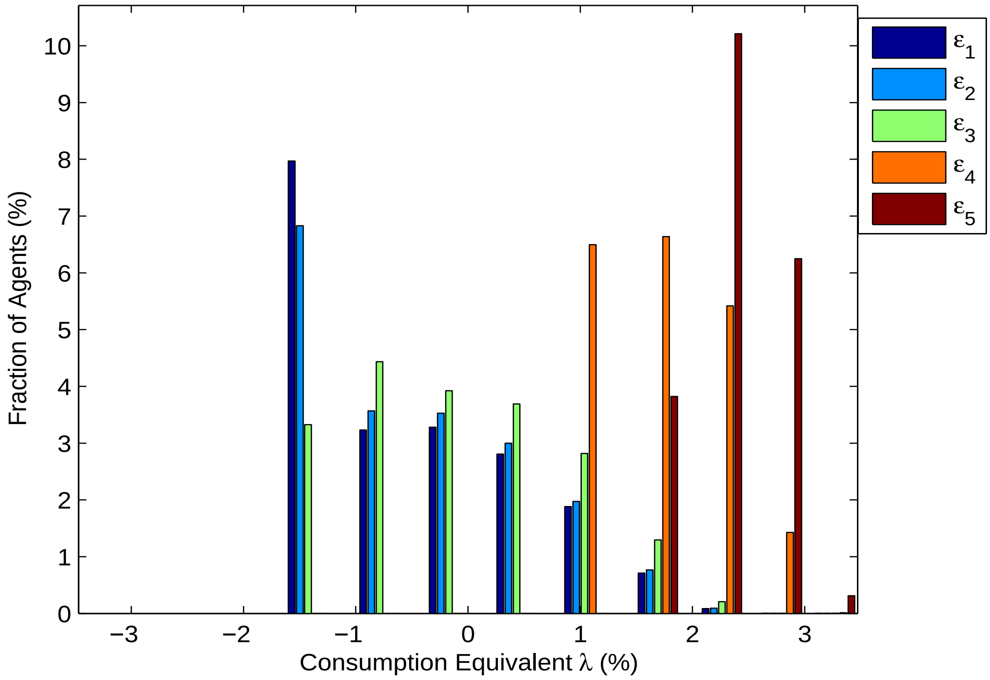


Figure 2.12: Histogram Welfare Gains $\lambda(k, \epsilon)$

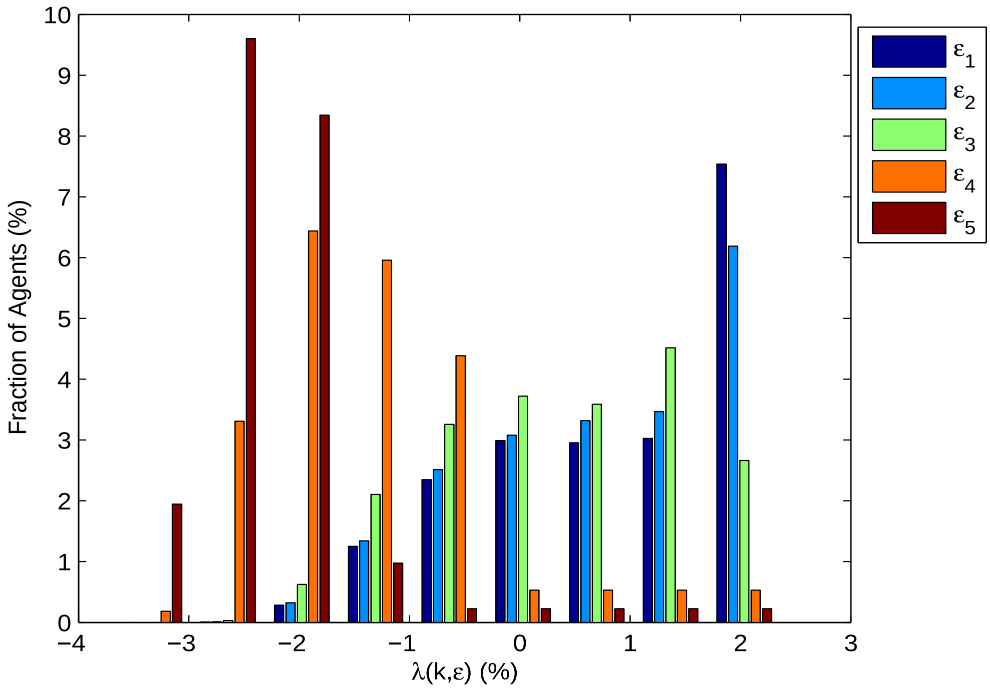


Figure 2.13: Histogram Welfare Gains $\lambda(k, \epsilon)$ by wage levels

2.7 Computational Algorithm

We now outline our algorithm for computing equilibria numerically. As in Krusell and Smith [75], we deal with the high dimensionality of the distribution by approximating Γ by a finite set of moments. One moment is the aggregate (or mean) capital stock K since this determines prices households face. The other moment is median after-tax income denoted γ defined by $(1 - \tau)[rk + w\epsilon]$ since this helps forecast the decisive voter and the evolution of the endogenous tax rate. Agents thus perceive the law of motion for K', γ'_m and τ' to be given by the functions $H(K, \gamma, \tau)$, $G(K, \gamma, \tau)$ and $\Psi(K, \gamma, \tau)$ respectively. Using this approximation we can re-formulate the household problem in an RCE as:

$$V(k,\epsilon,K,\gamma,\tau) = \max_{c,k'} u(c) + \beta \sum_{\epsilon'} \Pi(\epsilon'|\epsilon) V(k',\epsilon',K',\gamma',\tau') \tag{2.18}$$

s.t.

$$\begin{aligned} c + k' &= k + (1-\tau)\left[r(K)k + w(K)\epsilon\right] + T(K,\tau) \\ K' &= H(K,\gamma,\tau) \\ \gamma' &= G(K,\gamma,\tau) \\ \tau' &= \Psi(K,\gamma,\tau) \end{aligned}$$

The solution to this problem are the functions $h(k,\epsilon,K,\gamma,\tau)$ and $V(k,\epsilon,K,\gamma,\tau)$.

The one period deviation problem in (2.12) can be similarly redefined.

$$\widetilde{V}(k,\epsilon,K,\gamma,\tau,\tau') = \max_{c,k'}\ u(c) + \beta E_{\epsilon'|\epsilon}\left[V(k',\epsilon',\Gamma',\tau')\right] \tag{2.19}$$

s.t.

$$\begin{aligned} c + k' &= k + \left[r(K)k + w(K)\epsilon\right](1-\tau) + T, \\ K' &= \widetilde{H}\left(K,\gamma,\tau,\tau'\right), \\ \gamma' &= \widetilde{G}\left(K,\gamma,\tau,\tau'\right). \end{aligned}$$

The solution to this problem yields functions $\widetilde{h}(k,\epsilon,K,\gamma,\tau,\tau')$ and $\widetilde{V}(k,\epsilon,K,\gamma,\tau,\tau')$.

The distribution Γ is a probability measure on (S,β_S) where $S = [0,\bar{k}] \times E$ and β_S is the Borel $\sigma-$algebra. Thus, for $B \in \beta_S$, $\Gamma(B)$ indicates the mass of agents whose individual state vectors lie in B. For reference, here we also defined the operator $\Phi : M(S) \to M(S)$ where $M(S)$ is the space of probability measures on (S, β_S) :

$$(\Phi\Gamma)(k', \epsilon') = \int 1_{\{h(k,\epsilon,K,\gamma,\tau)=k'\}} \Pi(\epsilon'|\epsilon) d\Gamma(k, \epsilon). \tag{2.20}$$

An SSPRCE must be contained in the following set of stationary equilibria. Let $\tau_j \in \{\tau_1, ..., \tau_J\}$ be a grid of tax rates in $[0, 1]$ and let $\Gamma^{ss}(\tau_j)$ be an associated stationary distribution which solves RCE for $\tau' = \tau = \tau_j$. This procedure generates a set of stationary distributions and associated tax rates $SS = \{\Gamma^{ss}(\tau_j), \tau_j\}_{j=1}^{J}$. Simply put, this is like solving for the steady state of an Aiyagari [5] model for a grid of exogenous constant taxes.

1. Let $\Psi^n(K, \gamma, \tau)$ be the tax function at iteration n. For $n = 1$, we set this equal to a constant.

2. Given $\Psi^n(K, \gamma, \tau)$, solve a RCE. That is, let $H^s(K, \gamma, \tau)$ and $G^s(K, \gamma, \tau)$ be the functions associated with the law of motion for aggregate capital and median after tax income at iteration s. For $s = 1$ we set these to a constant.

   (a) solve for household decision rules (in particular saving $h^s(k, \epsilon, K, \gamma, \tau)$) in problem (2.18).

   (b) use the operator Φ defined in (2.20) and $\Psi^n(K, \gamma, \tau)$ to generate a joint sequence of transitional distributions Γ_η and tax rates τ_η for $\eta = 1, ..., \Upsilon$ starting from $\Gamma_0 = \Gamma^{ss}(\tau_j)$ and $\tau_0 = \tau_j$ for each of the $j = 1, ..., J$ possible tax rates. We take Υ large enough to ensure that $(\Gamma_\Upsilon, \tau_\Upsilon) \in SS$.

   (c) Use the J sequences of transitional distributions and taxes $\{\Gamma_\eta, \tau_\eta\}_{\eta=1}^{\Upsilon}$ to generate a sequence of $\{K_\eta, \gamma_\eta, \tau_\eta\}_{n=1}^{J\times\Upsilon}$. Run a linear regression on this sequence to update H^s and G^s as in Krusell and Smith [75]. If the updated H^s and G^s are close enough to the previous iteration, go to step 3, otherwise set $s = s + 1$ and go to step 2 with the updated functions.

3. Solve a PRCE.

(a) From step 2, we know $V(k,\epsilon,K,\gamma,\tau)$ which depends on $\Psi^n(K,\gamma,\tau)$ since it is in the constraint set in (2.18). Given this, we solve the one period deviation problem (2.19) starting from $\Gamma_0 = \Gamma^{ss}(\tau_j)$ and $tau_0 = \tau_j$ for each of $j = 1, ..., J$ in order to generates τ_1. Using the operator Φ evaluated at decision rules $\widetilde{h}(k,\epsilon,K_0,\gamma_0,\tau_0,\tau_1)$ obtain Γ_1 where K_0 and γ_0 are obtained from Γ_0. The next period's distribution and tax rate, (Γ_2,τ_2), are obtained by repeating the same steps starting at (Γ_1,τ_1). Continue in this way to compute the transitional sequence $\{\Gamma_\eta,\tau_\eta\}_{\eta=0}^{\Upsilon}$.

(b) Use $\{\Gamma_\eta,\tau_\eta\}_{\eta=0}^{\Upsilon}$ to generate the sequence $\{K_\eta,\gamma_\eta,\tau_\eta\}_{n=1}^{J\times\Upsilon}$. Run a linear regression on this sequence to update Ψ^n. If the updated Ψ^n is close enough to the previous iteration, go to step 4, otherwise set $n = n+1$ and go to step 1 with the updated functions.

4. Having solved for the functions H, G, and Ψ, solve for steady state K^*, γ^*, and τ^* that solves the three equations:

$$\begin{aligned} K^* &= H(K^*,\gamma^*,\tau^*) \\ \gamma^* &= G(K^*,\gamma^*,\tau^*) \\ \tau^* &= \Psi(K^*,\gamma^*,\tau^*). \end{aligned}$$

One-time voting simply restricts $\tau_\eta = \tau_1$ for all $\eta > 1$ in step 3a and uses (2.18) to generate the sequence $\{\Gamma_\eta,\tau_\eta\}_{\eta=0}^{\Upsilon}$ with $\tau_\eta = \tau_1$for all $\eta > 1$.

2.7.1 Computed Equilibrium

In this section we present the computed median voter sequential equilibrium for the Final Steady State calibration. We approximated the evolution of the wealth distribution on and off-the-equilibrium by a finite number of moments: mean capital, a measure of the median and the tax rate. In particular, the laws of motion we consider are:

- Law of motion of aggregate capital, function H

$$K' = a_0 + a_1 K + a_2 z_m + a_3 \tau \tag{2.21}$$

- Law of motion of median total resources, function G

$$z'_m = b_0 + b_1 K + b_2 z_m + b_3 \tau \tag{2.22}$$

- Law of motion of taxes, function Ψ

$$\tau' = d_0 + d_1 K + d_2 z_m + d_3 \tau \tag{2.23}$$

where

$$z_i = k + [r(K)k + w(K)\epsilon_i](1 - \tau) + T \tag{2.24}$$

In Table (2.7) we display the parameter values for the laws of motion. The equilibrium income effective steady state tax rate from this sequential equilibrium is 0.4562.

Variable	K'	z'	τ'
Constant	0.13	0.15	0.33
	(6.64e-08)	(5.54e-05)	(1.79e-04)
K	0.94	0.10	-0.10
	(1.08e-07)	(9.07e-05)	(3.22e-04)
z	-1.39e-02	0.78	0.16
	(1.79e-07)	(1.49e-04)	(5.28e-04)
τ	-6.63e-02	6.36e-03	0.12
	(1.19e-07)	(9.98e-05)	(3.70e-04)
R^2	0.999	0.998	0.892

Table 2.7: Equilibrium Laws of Motion

To illustrate the importance of using another moment like median resources, we solved the PRCE equilibrium without the law of motion (2.22) and with $a_2 = 0$ in (2.21) and $d_2 = 0$ in (2.23). Notice that the goodness of fit (measured by R^2) falls substantially for the law of motion of taxes (2.23) in Table (2.8).

Variable	K'	τ'
Constant	0.13	0.51
	(2.29e-06)	(7.83e04)
K	0.92	-1.64e-02
	(8.36e-07)	(2.83e-04)
τ	-8.19e-02	-0.18
	(1.11e-05)	(3.72e-03)
τK	1.51e-02	8.78e-02
	(4.07e-06)	(1.35e-03)
R^2	0.999	0.930

Table 2.8: Imperfect Equilibrium Laws of Motion

2.8 Concluding Remarks

At election time, the median voter mechanism assumes all agents vote. However, evidence shows that voter turnout varies across income quintiles. Table 2.9 displays voter turnout in the U.S. by income quintiles during the presidential elections of years 1980 and 1996 (the closest years to what we considered above). The values in the second and third columns of this Table correspond to the percentage of agents in each quintile that voted in a particular year. Notice that voter turnout is positively correlated with an agent's position in the income distribution and that there are not significant changes in observed voter turnout by quintile from 1980 to 1996.

Income Quintile q_i	1980 % of q_i	1996 % of q_i
q_1 (lowest)	54	52
q_2	61	57
q_3	63	65
q_4	71	73
q_5 (highest)	81	79

Table 2.9: Voter Turnout in Presidential Election by Income Quintile

While we do not have a model of voter turnout, here we simply consider what the observed voter turnout would imply for tax choices in our model. Specifically, since more rich people vote, one would expect that the equilibrium tax rate would reflect their numbers (i.e. we might expect lower taxes). Since our model overpredicts the average effective tax rate, this could in principle help to match the data.[22]

[22]That is, including a mechanism with more realistic weights (similar to those in Table 2.9), could help solve this problem by giving more power to voters in higher income quintiles. It is important to note that even if the above weights were consistent on-the-equilibrium path, off-the-equilibrium path weights could be very different from those in Table 2.9).

Chapter 3

Investment and Firm Dynamics

3.1 Introduction

Empirical studies have shown that firm size and growth are *not* independent for manufacturing firms in the U.S.. Evans (1987) and Hall(1987) show that the growth rate of employment of manufacturing firms, and the volatility of growth, are negatively related to firm size and age. Dunne et al. (1988) study U.S. manufacturing plants and show that the output of an entrant is considerably smaller than that of an average incumbent. Evans (1987) also finds that firm growth decreases with firm age and that this relation remains after conditioning on firms' size, and that firm growth decreases with firm size even when firms' age is held constant. Davis, Haltiwanger and Schuh (1996) show that the rates of job creation and job destruction in U.S. manufacturing plants are decreasing in age and size and that conditional on the initial size, small establishments grow faster than large firms. Thus, the empirical regularities[1] of firm dynamics are:

[1] Some of these empirical facts are shown using establishment data while others correspond to firm-level data. However, many of the empirical facts based on firm data also hold for single-unit establishments (i.e. establishments that are firms) and small establishments (see Evans (1987)). Moreover, a recent study by Rossi-Hansberg and Wright (2006) showed that the firm and establish-

(i) Firm growth decreases with firm age and size.

(ii) The variability of firm growth decreases with firm age and size.

(iii) Job creation and destruction[2] decrease with firm age and size.

(iv) **Size dependence and age dependence:**

- *Size dependence:* Conditional on age, the dynamics of firms (growth, volatility of growth and job creation and destruction) are negatively related to size;
- *Age dependence:* Conditional on size, the dynamics of firms (growth, volatility of growth and job creation and destruction) are negatively related to age.

It seems natural to link patterns of firm growth and job reallocation with their capital accumulation decision. In this paper, I ask whether a model of capital accumulation with adjustment costs and entry and exit, parameterized to match the investment regularities of U.S. manufacturing firms, is capable of generating size and age dependence. The main point of the paper is that this model is able to account for the conditional age and size dependence that Cooley and Quadrini (2001) model as arising from financial frictions.

Motivated by evidence showing that non-convexities and irreversibility play a central role in the investment process I extend a standard model of firm dynamics to include non-convex capital adjustment costs. The primary basis for this view is plant level evidence of a non-linear relationship between investment and measures of

ment size distributions are similar, reflecting the fact that only the very largest firms possess more than a single plant. This paper focuses on the technology of a single production unit and does not address questions of ownership or control.

[2]Following Davis et. at. (1996) job creation is defined as the sum of employment gains of expanding firms and job destruction is the sum of employment losses of contracting firms.

fundamentals, including investment bursts as well as periods of inaction[3]. Moreover, as plant size increases, investment expenditures become smoother.

The results of the paper can be summarized as follows. First, I show that in the stationary equilibrium a model of firm dynamics with entry and exit can capture the main investment characteristics of U.S. manufacturing firms. In particular, I observe that the model investment rate distribution has a considerable mass around zero, that smaller firms invest more and that as plant size increases, investment expenditures become smoother. I then show that the combination of a standard model of investment with adjustment costs and the introduction of entry and exit can generate the *simultaneous dependence* of industry dynamics on size (once we condition on age) and on age (once we condition on size).

In this paper, I argue that there is no conclusive evidence that financial frictions are a necessary condition to replicate the age and size dependence. I also show that a model, in which only friction is non-convex capital adjustment costs is capable of generating the relation between investment rates, a measure of the value of the firm (Tobin's Q) and profit rates observed in the data. Cooley and Quadrini (2001) were the first to capture the size and age dependence, linking the patterns of firm growth with financial frictions. However, the dynamics of their model are driven by the assumption that new entrants are of the highest productivity level, contradicting the fact that entrants are initially less capital-intensive and less profitable than incumbents. Moreover, in Cooley and Quadrini (2001), firms' capital dynamics are at odds with the investment behavior observed in the data.

The literature on capital accumulation has found that the standard assumptions of the neoclassical model of the firm, such as strictly convex adjustment costs and reversibility, fail to explain investment behavior adequately (see Abel and Eberly (1994, 1996), Caballero, Engel and Haltiwanger (1995), Caballero and Engel (1999),

[3]Cooper and Haltiwanger (2006) documented these facts at the plant level and Becker et. al. (2005) at a higher level of aggregation.

Cooper and Haltiwanger (2006), Cooper, Haltiwanger and Power (1999), Doms and Dunne (1994) for example). Motivated by the disappointing empirical evidence, other economists have argued in favor of the existence of non-convexities. The sources of the speculated non-convexities in the cost of capital adjustment include decreasing returns, the cost of equipment, costs associated with disruption and installation costs. The adjustment cost function in this paperincludes not only the traditional convex cost term but also a non-convex cost of investment associated with the level of profitability in periods of adjustment.

In this model, firms are characterized by their capital stock and productivity level. The optimal decision rules and the evolution of the idiosyncratic shocks generate an endogenous distribution of establishments across capital, productivity and age. The size dependence derives from the standard conditions of optimal investment and labor decisions in this environment, in which an abundance of capital leads to low rates of return and slower accumulation, while a relatively small stock of capital leads to higher returns and lower variability of future profits resulting in higher investment rates. Hence, small firms will grow faster than large firms. The age dependence is driven by the technological composition of firms in each age class. The distribution of entrants and the persistence of the productivity level play a decisive role. As a cohort of entrants gets older the persistence parameter defines how fast the distribution of these firms across shocks becomes equal to the stationary distribution. An initial distribution that differs from the ergodic distribution and a low persistence parameter increases the chances of the model of getting the right age dependence. We calibrate the stochastic process so that the model can reproduce the main investment facts and the relative size of entrants for U.S. firms.

Besides Cooley and Quadrini (2001), a number of authors have tried to explain the relation between size, age and firm dynamics as arising from persistent idiosyncratic shocks to firms' production technology or from learning by doing.

This literature includes the models studied by Jovanovic (1982), Hopenhayn (1992), Campbell and Fisher (2000), Alburquerque and Hopenhayn (2002), Clementi and Hopenhayn (2006) and Rossi-Hansberg and Wright (2006). These models can generate an *unconditional* dependence of the firm dynamics on size and age. In other words, as Cooley and Quadrini (2001) indicated, without conditioning on age, firm dynamics are negatively related to firm size, and without conditioning on size, firm dynamics are negatively related to firm age. However, they cannot account simultaneously for the *conditional* dependence on both size and age. My paper is also related to the earlier work of Castro, Clementi and Corbae (2005). Their paper tries to discriminate between two models of firm dynamics: (*i*) a learning model (symmetric and incomplete information) and (*ii*) a moral hazard model (asymmetric information). They assess whether informational frictions can successfully explain the conditional moments of firm dynamics in a model that also incorporates fixed and convex adjustment costs. Boyarchenko (2006) constructs a model of a competitive industry equilibrium refining the work of Dixit and Pindyck (1996) to study the implications of capital irreversibilities in continuous time where investment is made in several stages.

The rest of the paper is organized as follows. In Section (3.2) I describe the model and derive the conditions needed to find the stationary distribution. Section (3.5) presents the calibration and the computation of the model. In Section (3.6) I show the unconditional moments of firms' dynamics. Section (3.7) describes the main result of the paper, the size and age dependence. Finally, in Section(3.8) we conclude.

3.2 Environment

The environment is a simplified version of the model developed by Hopenhayn (1992) augmented to include capital accumulation and adjustment costs. The only source of uncertainty for firms currently in operation is the specific productivity shock. Incumbent firms maximize the expected present value of discounted profits and in every period decide the optimal production plan. The framework described below is designed for the purpose of studying a competitive economy that is in a stationary or long-run equilibrium. In this equilibrium some firms will be undergoing change over time, with some expanding, others contracting, some exiting the market and others starting up. Despite all these changes at the level of the individual firm, aggregate variables will be constant over time.

The firm[4] produces output y_t per time period with a production technology

$$y_t = f(s_t, k_t, n_t) = s_t k_t^{\alpha} n_t^{\gamma}, \tag{3.1}$$

with $\alpha \in (0,1)$, $\gamma \in (0,1)$, where s_t is the idiosyncratic productivity shock, k_t is the stock of capital that the firm employs in period t and n_t is the labor input. Realizations of the idiosyncratic productivity shock s take values in the set $S \equiv \{s_1, \ldots, s_{ns}\}$ with ns finite. The process of s_t is assumed to follow a First Order Markov Process with transition matrix $\Pi(s'|s)$ and to be iid across firms. This implies that there is no uncertainty over the aggregate state of the economy even though there is uncertainty at the individual level. Denote $\pi_{s',s} = Pr(s_{t+1} = s'|s_t = s)$ as the probability of receiving s' in period $t+1$ given that period t shock is equal to s. For each value of s, the vector $\Pi(\cdot|s)$ represents the distribution of future values of the shock, s'. It is assumed (as in Cooley and Quadrini (2001)) that active firms face a probability of receiving a shock $s_t = 0$ denoted by π_x. Moreover, once s_t reaches this value there is zero probability that s_t will receive a positive value in the future. Given these assumptions it is natural to identify a zero value for the productivity shock with the death of a firm.

[4]Through the paper I consider single-unit firms, i.e. plants/establishments that are firms. See footnote 1.

3.2.1 Incumbent Firm's

The operative profits of an active plant are given by

$$P(s_t, k_t, n_t) = f(k_t, n_t, s_t) - wn_t \tag{3.2}$$

After observing the productivity shock and making the labor decision, every continuing plant decides the optimal level of investment

$$i_t = k_{t+1} - (1-\delta)k_t.$$

We normalize the price of new capital to 1 and denote the selling price of capital by p_s. Following the literature on plant dynamics, we assume that to modify the level of capital the plant must incur adjustment costs. The function $g(k_t, k_{t+1})$ captures the presence of these costs and is defined as follows:

$$g(k_t, k_{t+1}) = \begin{cases} (1-\lambda)P(k_t, n_t, s_t) + \frac{\psi}{2}\left(\frac{i_t}{k_t}\right)^2 k_t, & \text{if } i_t \neq 0, \\ 0, & \text{if } i_t = 0; \end{cases}$$

For values of $i_t \neq 0$, the first term in $g(k_t, k_{t+1})$ captures the disruption costs associated with the installation of new capital. A fraction $\lambda \in (0, 1)$ of the operating profits is lost in the period of adjustment. Empirical studies (see for example Power (1998)) provide evidence that plant productivity is lower during periods of large investment. Note that ceteris paribus, investment rates are lower in periods of low productivity. The last term is the traditional convex adjustment cost.

The establishment's objective is to maximize the discounted present value of profits by choosing the optimal level of investment. The timing within period t for a plant that produced in period $t-1$ is as follows: First, the exit shock is realized. If the firm has to exit, it collects its capital and stops producing for ever. If not, the idiosyncratic productivity level is realized. Second, active plants decide the optimal level of labor input and the investment decision. Before the period ends, the firm receives profits net of adjustment costs (if any).

3.2.2 Entry Decision

I assume that there is a continuum of ex-ante identical potential entrants in each period. Entrants incur a one time fixed cost κ_e denominated in units of output. The price of capital at the entry stage is denoted by c_e. Each potential entrant receives its initial shock from a continuous distribution $\nu(s)$. The size and the distribution of entrants will play an important role in the dynamics of firms. The determinants of this relation will be explained in more detail later.

It is assumed that in this economy there is free entry. The timing of events before entry is as follows: First, the potential entrant observes the costs of creating a new firm and then decides to enter or not. Second, if he decides to enter, the entrepreneur pays the fixed entry cost and makes the initial investment k^e.

3.3 Recursive Formulation

3.3.1 Incumbent Firm's Problem

For any firm with $s \in S$ the optimal level of labor input solves the following problem:

$$\max_n \left\{ sk^\alpha n^\gamma - wn \right\} \tag{3.3}$$

The solution implies that the optimal labor choice at state (s, k) is:

$$n(k, s) = \left[\frac{\gamma s k^\alpha}{w}\right]^{1/(1-\gamma)}.$$

Define $a = (s/w^{\gamma})^{\frac{1}{1-\gamma}} \left[\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}\right]$ and $\theta = \frac{\alpha}{1-\gamma}$. Then, by evaluating the profit function $P(k, n, s)$ at the optimal labor decision $n(k, s)$, I can derive the operating profit of the plant (as a function of k and a), $R(k, a)$, as follows:

$$R(k, a) \equiv P(k, n(k, s), s) = ak^{\theta}. \tag{3.4}$$

For notational simplification and ease in the exposition, I will write the problem of the firm as a function of the variable a. The transition probabilities will be denoted by $\Pi(a, a')$ and the entrant probability distribution by $\nu(a)$. The curvature of the profit function, θ, is particularly important for obtaining size dependence.[5] For future reference, it is convenient to redefine the optimal labor decision as a function of the variable a. Let $\phi = (\frac{\gamma}{1-\gamma})\frac{1}{w}$. After some simple algebra, I derive that optimal labor at state (k, a) is:

$$n(k, a) = \phi a k^{\theta}. \tag{3.5}$$

The recursive problem of the active plant is:

$$V(k, a) = max\left\{V^b(k, a), V^s(k, a), V^i(k, a)\right\} \tag{3.6}$$

where $V^b(k, a)$ represents the value of "buying" more capital, $V^s(k, a)$ corresponds to the value of "selling" capital and finally $V^i(k, a)$, inaction, is the value of keeping the depreciated capital stock for the future period.

[5]This profit function can also be derived from an environment where the firm faces a downward-sloping demand function and has a Cobb-Douglas constant return to scale production function.

The value of buying capital is:

$$V^b(k,a) = \max_{k' \in (k(1-\delta),\bar{k}]} \Bigg\{ R(k,a) - i - g(k,k') \tag{3.7}$$
$$+ \frac{1}{1+r}\Big[(1-\pi_x)\sum_{a'} V(k',a')\Pi(a,a') + \pi_x p_s k'\Big]\Bigg\}.$$

The value of selling capital is:

$$V^s(k,a) = \max_{k' \in [0,k(1-\delta))} \Bigg\{ R(k,a) - p_s i - g(k,k') \tag{3.8}$$
$$+ \frac{1}{1+r}\Big[(1-\pi_x)\sum_{a'} V(k',a')\Pi(a,a') + \pi_x p_s k'\Big]\Bigg\}.$$

where $p_s \leq 1$ represents the selling price of capital.

Finally the value of inaction is given by:

$$V^i(k,a) = R(k,a) + \frac{1}{1+r}\Big[(1-\pi_x)\sum_{a'} V(k(1-\delta),a')\Pi(a,a') + \pi_x p_s k(1-\delta)\Big]. \tag{3.9}$$

Note that in this last case the future value of capital is given by the depreciated capital stock after production in the current period.

3.3.2 Entry Decision

Entrants incur in a one time fixed cost κ_e denominated in units of output. The price of capital at the entry stage is denoted by c_e. Each potential entrant receives its initial shock from a continuous distribution $\nu(a)$. The initial investment k^e is the solution to:

$$k^e = \arg\max_{k'} \left\{ \frac{1}{1+r}\sum_{a'} V(k',a')\nu(a') - c_e k' - \kappa_e \right\}. \tag{3.10}$$

For future reference, I define the value of creating a firm as follows:

$$V^e \equiv \frac{1}{1+r} \sum_{a'} V(k^e, a')\nu(a') - c_e k^e - \kappa_e. \tag{3.11}$$

In this partial equilibrium analysis, with fixed prices, the mass of new entrant firms is non-degenerate only if the surplus from creating a new firm is zero, that is:

$$V^e = 0 \Leftrightarrow \frac{\sum_{a'} V(k^e, a')\nu(a')}{1+r} = c_e k^e + \kappa_e. \tag{3.12}$$

If the last condition holds with equality an equilibrium with positive measure of entrants will exist. By the properties of the value function that solves problem (3.6), the solution to (3.10) exists and it is unique. I do not conduct a general equilibrium analysis, but the entry of new firms will induce changes in prices (in particular in the wage rate) and in the value of the firm until there are no gains from creating a new firm.

I will focus the attention in the stationary distribution to study the long run properties of the model with adjustment costs. The stationary equilibrium implies a size and age distribution of firms. We provide conditions under which the empirical regularities hold.

3.4 Stationary Distribution

The only uncertainty in the model is generated by the idiosyncratic productivity shocks. At each point in time t the economy is characterized by a measure of firms $\Gamma_t(k, a, j)$ for each level of capital stock $k \in \mathbf{K} = [0, \bar{k}]$, productivity shocks $a \in \mathbf{A} = \{a_1, \ldots, a_{na}\}$ and age of the firms $j \in \mathbf{\Upsilon} = \{1, 2, 3, \ldots\}$. A discussion of the definition of the set $\mathbf{K}$ is in order. We will look for a stationary measure of firms, and this requires that firms never accumulate capital beyond some endogeneously determined level $\bar{k}$. Intuitively the value of $\bar{k}$ is where the decision rule $k'(k, a_{na})$ crosses the 45° line, provided that the optimal capital accumulation rule is an increasing function of a. Conditions under which firms optimally decide to do this are given in the quantitative section.

With a positive probability of receiving $a = 0$ in any given period, the expected age of exit is finite. If we let the measure of firms at age j be given by μ_j, then $\mu_{j+1} = (1 - \pi_x)\mu_j$, where the measure μ_0 is given and corresponds to the mass of new entrants.

Let $\mathbf{B}(\mathbf{K})$ and $\mathbf{B}(\boldsymbol{\Upsilon})$ be the Borel $\sigma-$algebra of $\mathbf{K}$ and $\boldsymbol{\Upsilon}$ respectively, and $\mathbf{P}(\mathbf{A})$ the power set of $\mathbf{A}$. Define $\mathbf{X} = \mathbf{K} \times \mathbf{A} \times \boldsymbol{\Upsilon}$. Let $\mathcal{X} = \mathbf{B}(\mathbf{K}) \times \mathbf{P}(\mathbf{A}) \times \mathbf{B}(\boldsymbol{\Upsilon})$ and $\mathbf{M}$ be the set of all finite measures over the measurable space $(\mathbf{X}, \mathcal{X})$.

The law of motion of $\Gamma_t(k, a, j)$ is given by:

$$\Gamma_{t+1} = H_t(\Gamma_t), \tag{3.13}$$

where the function H_t can be written explicitly as:

a. For all $\mathcal{T}$ such that $1 \notin \mathcal{T}$:

$$\Gamma_{t+1}(K \times A \times \mathcal{T}) = \int M_t((k, a, j); K \times A \times \mathcal{T}) \;\; \Gamma_t(dk \times da \times dj), \tag{3.14}$$

where

$$M_t((k, a, j); K \times A \times \mathcal{T}) = \begin{cases} \Pi(a, a')(1 - \pi_x) & \text{if } k'(k, a) \in K \\ 0 & \text{else} \end{cases}$$

b.

$$\Gamma_{t+1}(K \times A \times \{1\}) = \begin{cases} \nu(a)E & \text{if } k^e \in K \\ 0 & \text{else} \end{cases}$$

where E corresponds to the mass of entrants.

The explicit formulation of the law of motion for the distribution has to be divided in two parts in order to capture the assumption that entrant firms start their lives with capital value k^e.

This paper focuses on the study of the invariant distribution of firms denoted by Γ^*. We find Γ^* as the fixed point of this mapping, that is, $\Gamma^* = H(\Gamma^*)$. We normalize the measure of firms to one. The mass of entrants, E, will coincide with the mass of firms that exit the market, i.e. $E = \pi_x \Gamma^*$. In this way the total mass of firms is constant. Stokey and Lucas (1989) state the necessary conditions for convergence of the measure Γ. The properties of the stochastic process and the decision rules give rise to a mapping from the current distribution to the next period measure of firms. An invariant measure of firms Γ^* exists. Moreover, Γ^* is unique, and the sequence of measures generated by the transition function, $\{H^n(\Gamma_0)\}_{n=0}^{\infty}$ converges weakly to Γ^* from any arbitrary Γ_0. This result will allow me to calibrate the model using the stationary distribution and the moments from data on the U.S. manufacturing sector to then test the model implications for the conditional size and age dependence.

3.5 Matching Investment Moments

In this section, I parameterize the model to match the investment dynamics observed in the U.S. manufacturing sector. I assume that a model period is one year. To solve the firm's problem, I approximate the value function using cubic splines and the optimal capital accumulation is obtained by a root finding algorithm. I assume that the firm's idiosyncratic shocks (defined as in equation (3.4)) follow an autoregressive form given by

$$\ln(a_{i,t}) = \rho_a \ln(a_{i,t-1}) + u_t \tag{3.15}$$

with $u_t \sim N(0, \sigma_u)$ and $|\rho| < 1$. Denote the standard deviation of $\log(a)$ by $\sigma_a = \sigma_u / \sqrt{(1-\rho^2)}$. To solve the model, I will approximate the distribution of the idiosyncratic shocks using the method proposed by Tauchen and Hussey (1991) and choosing the number of grid points for a to be equal to 21.

The distribution of entrants and their initial level of investment are key elements in the analysis. I assume that the entry shocks are distributed in the same way as the innovation term in 3.15. In particular, $\nu(a)$ is the log-normal distribution, i.e. the log of an entrant's productivity shock, $\ln(a)$, is distributed $N(0, \sigma_u)$. The parameters of the productivity process can be chosen to match the profile of US firms. They have implications for the degree of persistence and dispersion in the distribution of firms. The size of entrants is determined by the entry costs and the distribution of entry shocks. The general pattern observed is that entering firms are substantially smaller on average than existing or continuing firms and that organization learning appears to continue over a period of at least 10 years.

The set of parameters necessary in order to compute the model are:

$$\Theta = \{\delta, \theta, \gamma, \rho_a, \sigma_a, \lambda, p_s, \psi, c_e, \phi\}, \tag{3.16}$$

where δ is the depreciation rate, θ is the curvature of the production function, γ is the corresponding labor share, ρ_a and σ_a are the parameters that define the idiosyncratic shocks, λ is the parameter that captures the disruption costs associated with capital adjustment, p_s is the relative price of used capital to new capital, ψ is the weight in the convex adjustment cost, and c_e is the corresponding variable entry cost.[6] Since the productivity process estimated is that of the variable a, in what follows, the value of the entry cost κ_e is such that the arbitrage condition 3.12 is satisfied and ϕ is set such that the average number of workers in the economy is

[6] Recall that $\theta = \frac{\alpha}{1-\gamma}$, so after setting the parameters θ and γ, the value of α can be derived as $\alpha = \theta(1-\gamma)$.

consistent with the data. This implies that ϕ is chosen so that the average number of workers in an establishment in the economy,

$$\overline{N} = \sum_j \int_{K \times A} n(k,a)\Gamma(dk,da,j) = \phi \sum_j \int_{K \times A} ak^{\theta}\Gamma(dk,da,j),$$

is equal to 65 as reported by Davis and Haltiwanger (1990). Once I set the value of ϕ, for a given γ, the wage rate w can be determined.

The exercise consists of choosing the parameters in order to match the long-run moments from the U.S. economy and the investment dynamics to then test the model against the size and age dependence moments. I set the risk free rate to 4 percent which corresponds to the average yearly real return on a five year T-bill sin 1980. I set the depreciation rate to be 0.11. Also, the labor-share parameter $\gamma = 0.64$ is in turn selected to replicate the labor share in the NIPA.

The exit probability is calibrated to $\pi_x = 0.045$ because in the sample analyzed by Evans (1987), the average probability of exit is about 4.5 percent.

To find the remaining parameters it is necessary to solve the model. The set of moments and parameters are jointly determined. The calibration of the curvature of the production function (3.4) is not straightforward in the presence of adjustment costs that interact with the profit function. Following Cooper and Haltiwanger (2006), a two step procedure is used to calibrate the value of θ that is consistent with the observed curvature of the production function at the plant level in the data. Using the assumed productivity shocks, equation (3.15), taking logs of the revenue function, equation (3.4) and quasi-differencing yields

$$\log(R_t) = \rho_a \log(R_{t-1}) - \rho_a \theta \log(k_{t-1}) + \theta \log(k_t) + u_t. \tag{3.17}$$

Cooper and Haltiwanger (2006) estimated this equation via generalized method of moments using a complete set of time dummies and lagged and twice-lagged capital and twice-lagged profits as instruments. The data come from the Longitudinal Research Database consisting of approximately 7000 large manufacturing plants that were in operation between 1972 and 1988. Their estimate of θ is 0.592. The two step procedure consists of setting θ, ρ_a and σ_a at some value and then calibrating the other parameters to match the corresponding moments. After this is done, the model is simulated to estimate equation (3.17) to recover the value of θ and ρ_a that is consistent with active establishments in the model. If the estimates coincide with the values of the parameters in the actual data I stop, if not I update θ and repeat the process. The parameters ρ_a and σ_a are taken directly from the Cooper and Haltiwanger (2006) estimation with disruption costs and after controlling for a time fixed effect.

The parameters associated with the adjustment cost function (λ and ψ), the selling price of capital (p_s) and the entrants' price of capital (c_e) are chosen so plants in the stationary distribution display the patterns documented in Cooper and Haltiwanger (2006) (investment facts) and Dunne, et. al. (1988) (entrant's main characteristics). The main findings of Cooper and Haltiwanger (2006)can be summarized as follows: first, plants exhibit significant inaction in terms of capital adjustment (8.10 % of the total observations have investment rates of less than 1% in absolute value). Second, periods of inaction are complemented by periods of rather intensive adjustment of the capital stock. Cooper and Haltiwanger (2006) (and many others) define a spike as an investment episode in excess of 20 %. Negative spikes are found in 1.8 percent of the observations. The correlation between investment rates and profitability shocks is 0.143 in the data. The model does not include aggregate shocks and these moments are averages across time and affected by business cycle fluctuations. However, these movements are found to be small.

Dunne et. al. (1988) report that entrants that create a firm by building a new plant have market share of 10.4% and their relative size is 28.35% of the average firm.[7][8] This study summarizes the patterns of firm entry, growth and exit in four-digit U.S. manufacturing industries over the period 1963-1982. Entrants are disaggregated into new firms, existing firms that diversify into an industry by opening new production facilities, and existing firms that enter by altering the mix of outputs they produce.

The full set of parameter values is reported in Table (3.5). The curvature of the production function θ is equal to 0.604. This implies that the capital share α is equal to 0.2174 (Fuentes, Gilchrist and Rysman (2006), Gomes (2001) and Hennessy and Whited (2005) obtained similar estimates[9] in related studies). The selling price of capital is smaller than the price of buying new capital by approximately 1%. This wedge between the prices is in part responsible for the inaction that the model generates. The disruption cost is around 15% of current profits in periods of adjustment. Given the value of ψ, firms with high enough levels of capital will find it optimal to reduce the scale of production and do not wait until the depreciation process takes all the excess. Moreover, given the combination of adjustment costs present in the model, firms will wait until the productivity shock is high enough to increase the capital stock up to the optimal level and we will observe the bursts of investment that are documented in the literature. To have a better sense of the magnitude of these parameters, the average adjustment cost paid relative to the capital stock was 4% in the stationary distribution. The calibrated value of c_e is 1.41 and implies that the cost of buying new capital is around three times of the

[7]Their study is based on Census data that is available every 5 years. For that reason the market share in the model corresponds to the ratio of total output produced by firms of age 1 through 5 over total output of older firms.

[8]The relative size is computed as the ratio of average output of entrants over average output of older firms (see previous footnote).

[9]This parameter value also produces an equilibrium capital-output ratio consistent with the US economy when the empirical counterpart for capital is identified with plant and equipment and is associated in NIPA with nonresidential investment.

price active plants face.

Parameter		Value	Moment	Value
Discount Rate	r	0.04	U.S. Data	
Depreciation Rate	δ	0.07	LRD Plants	
Labor Share	γ	0.64	NIPA	
Exit Probability	π_x	0.045	Observed exit probability	
Autocorrelation	ρ_a	0.885	LRD Plants	
Std. Dev. of a	σ_a	0.64	LRD Plants	
Capital Share	θ	0.604	Establishments LRD	
Disruption Cost	λ	0.882	Average Investment Rate	12.2%
Selling Price	p_s	0.988	Fraction Spike (-)	1.8%
Convex Cost	ψ	0.112	Corr (i, a)	14.3%
Entry Cost	c_e	1.41	Entrants Market Share	10.4%
	ϕ	9.87	Average Employment	62

Table 3.1: Model parameters

3.6 Firm Dynamics and Stationary distribution

In this section, I describe the firm dynamics generated by the calibrated model of adjustment costs at the stationary distribution. The intuition behind firm's behavior is provided in Figure 3.1 which shows the value of the firm, the optimal investmet decision rule, the labor choice and Tobin's Q for different combinations of firm's capital size and idiosyncratic shocks.

The value of the firm is strictly increasing in firm's size (capital). As a result of decreasing returns to scale, the marginal increase in value is decreasing in k. This implies that for low values of k the marginal benefit of investment is higher and the firm will prefer to invest in new capital. On the contrary, everything else equal, for high values of k it will prefer to sell some of its capital stock and reduce the scale of production.

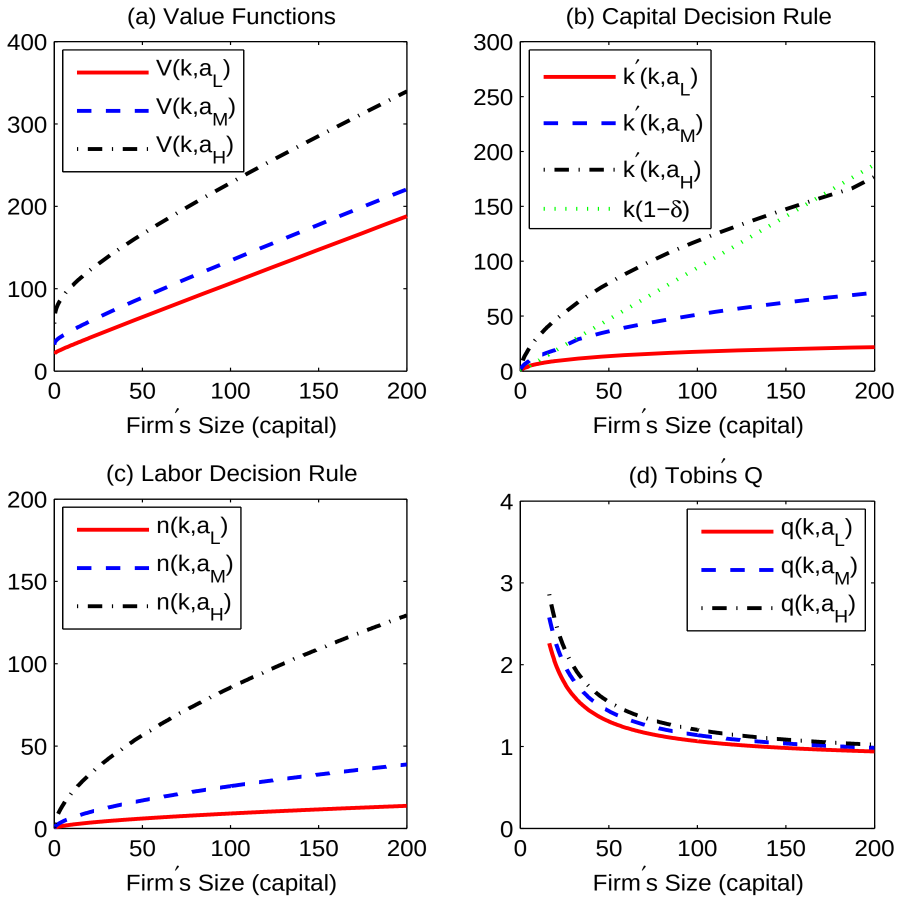


Figure 3.1: Firm Behavior

Figure 3.1.b shows the optimal capital accumulation of the firm for low (a_L), medium (a_M) and high (a_H) values of a. We observe that $k'(k,a)$ is strictly increasing in k and s. For low values of k, $k'(k,a) > k(1-\delta)$ that is, $i > 0$. There are middle range values of k where $k'(k,a)$ coincides with $k(1-\delta)$. In this case the

combination of k and s are such that the firm prefers not to invest. Finally, for high values of k, investment is negative, that is the firm is selling some portion of its capital at price p_s when the marginal product of one extra unit of capital is less than the marginal benefit of selling it. This pattern can be associated with two investment thresholds: one that defines when to stop investing and set $k' = (1 - \delta)k$ and another that determines when to start selling capital. This thresholds are increasing in the productivity shock of the firm for a given current's capital stock k. Another feature that we can observe from this figure is the endogenous determination of $\bar{k}$.

Small firms will invest in new capital as they move from low to high productivity shocks; that is, firms increase their capital stock when their future prospects increase. Firm behavior implies that the investment rate of firms is sensitive to the profit rate, $R(k, a)/k$, even after controlling for the future profitability of the firm and this sensitivity is greater for smaller firms (see section 3.7.1 for a deeper explanation). As pointed out in previous literature, it is possible to obtain cash-flows effects even in the absence of financial frictions. The labor decision rule is also depicted in Figure (3.1). This decision rule comes directly from equation (3.5). Firm's labor choice is strictly increasing in its capital and its productivity shock.

Now I turn to the properties of firms' dynamics in the stationary distribution. I calibrate the model mainly to match the moments reported by Cooper and Haltiwanger (2006) and the distribution of entrants presented in Dunne, Roberts and Samuelson (1988, page 504). The histogram of investment rates that emerges from this economy is reported in Figure(3.2). Clearly, there is a mass of firms around zero investment as we observe in the data (see Figure 1 in Cooper and Haltiwanger (2006) for example) and the distribution has fat tails.

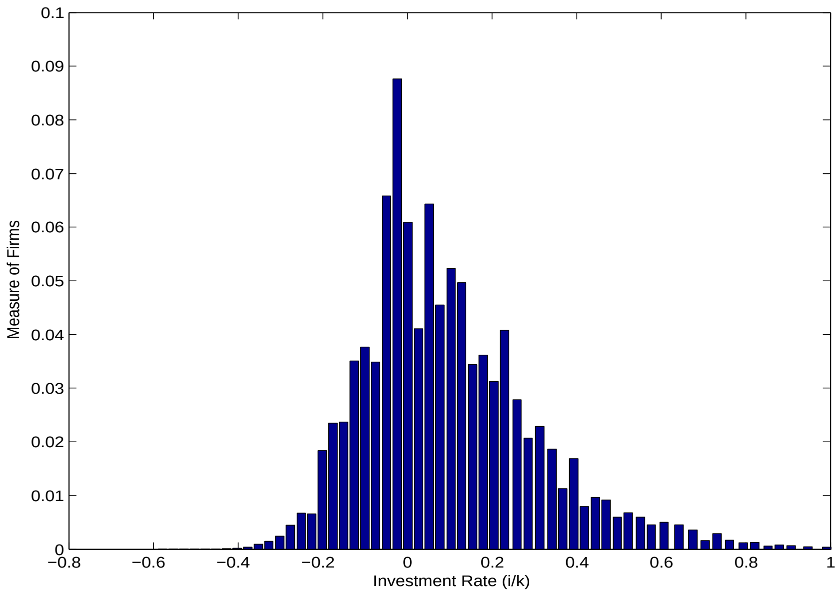


Figure 3.2: Model's Distribution of Investment Rates

The moments from the data and those from the model in the stationary distribution are reproduced in Table (3.2). At the calibrated parameter values the model does a good job in reproducing the investment moments.

Moment	Data	Model
Spike Rate: Negative Investment (%)	1.80	1.78
Inaction Rate (%)	8.10	8.12
Correlation($I/K, a$)	0.143	0.143
Entrant Relative Size (%)	28.35	28.33
Entrant Market Share (%)	10.4	10.4

Table 3.2: Data and Model Investment Moments.

As displayed in Figure (3.3), I also explore the properties of the growth rate of capital, profit rate, the standard deviation of the growth rate and job reallocation rate dynamics to show that they are consistent with the observations in the U.S. economy. These unconditional moments are computed by averaging them according to the stationary distribution of each class of firms. At a given age, firms differ in two dimensions: their capital stock and their productivity shock. This heterogeneity is the driving force of all my results.

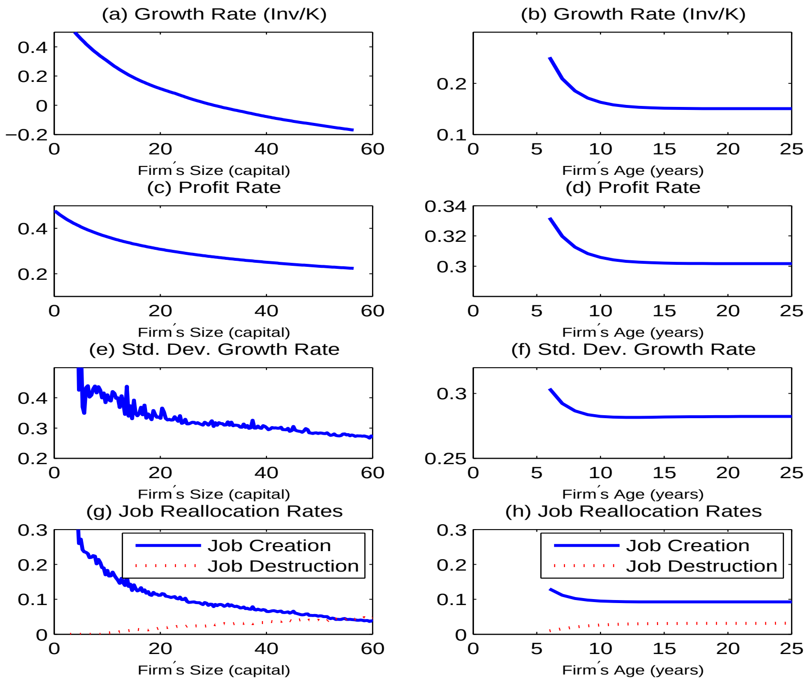


Figure 3.3: Firm's Moments and Invariant Distribution

The key properties of the behavior of firms that can be summarized as follows:

- Small and younger firms growth faster (Panels (a) and (b)).
- Profits rates are negatively correlated with size and age (Panels (c) and (d)).
- The variability of firm growth decreases with firm age and size (Panels (g) and (h)).

- Small and younger firms have higher job reallocation rates (Panels (e) and (f)).

The growth rate of capital is plotted in Panels (a) and (b). To understand why investment rates are a decreasing function of capital, it is necessary to understand the trade-off that firms face when deciding the optimal level of capital for the future. On the one hand, more capital allows them to increase the production scale and increase their expected profits; on the other hand the expansion of the production scale combined with the capital irreversibility implies a higher exposure to the bad productivity shock. Naturally, this is a result of the diminishing returns to scale at the establishment level. At higher levels of capital the expected profits of increasing the size of the firm decreases. The investment behavior will be important in explaining the relation between growth rates and cash flows. The unconditional dependence of firm growth rates on their age comes from the fact that firms are born small on average. As they become older they also become bigger and the intuition is similar to the unconditional dependence of firms' growth rate on size.

Panels (c) and (d) plot profit rates as a function of firm's size and firm's age. This property derives also from the decreasing return to scale production function and the optimal capital accumulation rule mentioned above. The higher profitability of smaller firms implies that they have a greater incentive to reinvest profits resulting in the pattern of investment rates observed in Panel (a). Similarly, the relation between panel (d) and panel (b) is the driving force of the unconditional dependence of profit rates on age.

The standard deviation of growth is also a decreasing function of size (except for a range of small firms and age (see Panels (e) and (f)). Smaller (and younger) firms behavior are more affected by productivity shocks than bigger (and older) firms.

Panels (g) and (h) display the job creation rate, defined as the rate of employment gains summed over all plants that expand at a given age or size category (see Davis, Haltiwanger and Schuh (1996)), and the job destruction rate, defined as the rate of employment losses summed over all plants that contract or shut down. As pointed before, the labor decision is increasing in capital, so the dynamics of firms' growth stated above have a direct effect on the job creation rate of labor. The model is capable of generating the unconditional dependence of firms' reallocation on size and age. Job reallocation, i.e. the sum of job creation and destruction is decreasing in size and age; however, while it is possible to observe that job creation is decreasing in size and age, job destruction is increasing for young and small firms.

Finally, Figure (3.4) plots the joint distribution of firms over size (capital stock) and age. New entrants are of the same size; however, they make different investment decisions according to their productivity shocks in their first period of life. In the model studied by Cooley and Quadrini (2001) entrants are always of the highest productivity shock. We observe a concentration of small and young firms. This is a feature of the model that is consistent with the data even though the model is calibrated only to match the investment facts. In the U.S. manufacturing sector, more than $\frac{4}{5}$ of new plants exit within 20 years. Furthermore, we observe that younger firms are smaller on average.

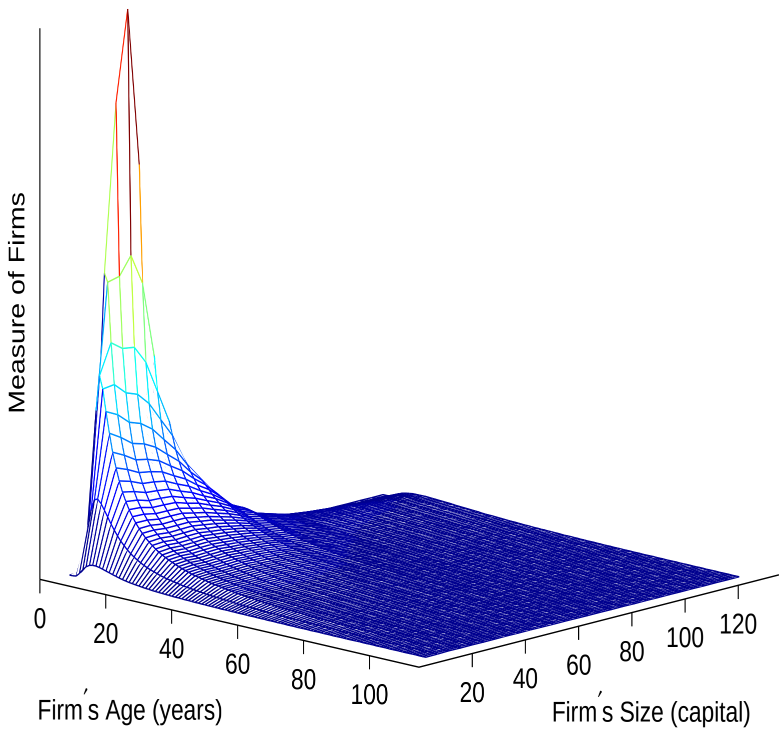


Figure 3.4: Stationary Distribution of Firms over Size and Age.

In summary, once calibrated to match the investment features of U.S. establishments, the model is able to generate the *unconditional* size and age dependence of firms' dynamics. Now is time to see if the model with adjustment costs and idiosyncratic productivity shocks is also able to generate simultaneously the conditional size and age dependence. Furthermore, in Section 3.7.1, I will show that the model is consistent with the evidence that relates investment rates and cash flows.

3.7 Size and Age Dependence.

The analysis conducted in the previous section showed that a model driven by productivity shocks, with convex and non-convex adjustment costs, some level of capital irreverisibility and entry and exit calibrated to the U.S. data captures many of the salient qualitative features of industry dynamics. In particular, higher investment rates, higher profits, higher volatility and higher job reallocation are observed for small and young firms. However, the main point of the paper is to demonstrate that this model is also able to account for the conditional age and size dependence pointed by previous studies like Evans (1987), Hall (1987) and Davis et al. (1996) that Cooley and Quadrini (2001) describe as arising from financial frictions.

Previous models of industry dynamics that consider investment decisions were not able to generate the age and size dependence because, once you control for the size of the firm, age becomes irrelevant in differentiating the dynamics of small and large establishments: the dependence on age derives only from the fact that young firms are on average smaller. In those models there exists only one dimension of heterogeneity, and thus once you fix age or size, firms are all alike independently of their history. In this model, there exist two dimensions of heterogeneity, because once you condition on size (capital stock depends on the previous history), firms could also be different in the productivity composition. Furthermore, once you condition on the level of the idiosyncratic shock, firms differ in their size and this generates different patterns for the capital stock. To analyze how the model behaves I create a panel of firms from the stationary distribution[10], and I conduct an econometric test to verify the relation between growth rates, profits rates, job creation and job destruction with the size and the age of the firms. Specifically, for each of these four variables, denoted by x, I run the following regression[11] on the

[10] In each simulation, I simulate 10,000 firms for 2,000 periods and take the last 200 periods.

[11] I also tried a specification with higher order terms and the results were similar.

simulated data

$$x_{j,t} = a_0 + a_1 \ln(size_{j,t}) + a_2 \ln(age_{j,t}) + \epsilon_t. \tag{3.18}$$

The subscript j denotes firm j and t corresponds to the time period. As before, firm's size is measure with the stock of capital. The results of this test are displayed in Table (3.3).

	Growth Rate	Profit Rate	Job Creation	Job Destruction
Constant	2.0872	0.7558	0.2800	0.2785
	$(1.32e-03)$	$(2.11e-04)$	$(2.32e-04)$	$(5.85e-03)$
Size	-0.2806	-0.0371	-0.0096	-0.0463
	$(4.87e-05)$	$(9.29e-05)$	$(1.49e-06)$	$(1.73e-05)$
Age	-0.3350	-0.1070	-0.0431	-0.0391
	$(1.18e-04)$	$(5.67e-05)$	$(7.24e-05)$	$(8.26e-06)$

Table 3.3: Model Predictions (Std Dev in parenthesis)

The elasticities of growth, profit rates, job creation and job destruction with respect to size and age are negative. Therefore, firm growth profit rates, job creation and job destruction decrease with firm size when firm age is held constant and decrease with firm age when size is held constant. Every decision of the firm depends on its level of capital stock as well as its productivity shock. Two firms with the same productivity shock will decide to invest, disinvest or continue with the same scale of production according to their level of k. Different values of k reflects the different histories. Similarly, two firms with the same scale of production will invest different amounts of capital according to their current value of s. The heterogeneous behavior of firms in the stationary distribution introduces the age and size dependence.

Even though the model was parameterized only to match the investment moments it reproduces also the quantitative results in the literature relating firms dynamics conditional on size and age. Evans (1987) finds that the elasticity of growth rates with respect to size is -0.0374 and with respect to age is -0.0381. That is, as in the model, firm growth decreases with firm size when firm age is held constant and decreases with firm age when firm size is held constant.

The conditional size dependence derives from the same factors that affected the unconditional relation between firms' evolution and size. The higher growth rates of small firms is related with their higher profit rates and higher value of Tobin's q. This also introduces a negative relation with the rates of job reallocation (creation and destruction). As explained in the previous section, this behavior is directly connected with the decreasing returns to scale production function. Smaller firms perceive a higher marginal benefit of investing in new capital and consequently their growth rates are higher. A similar intuition applies for the rest of the variables.

The age dependence is driven by the heterogenous technological composition of firms of different ages classes. Conditional on their size, firms with higher productivity shocks experience higher rates of profits than firms with lower values of s. Conditional on the size of the firms, the distribution across productivity shocks differs as we move from younger to older firms. As a result, younger firms grow faster and face higher rates of job creation and failure than older firms.

Using the results from Table 3.3, in Figures (3.5) and (3.6) I plot the growth rate of firms, their profit rate, job creation and job destruction rates as functions of the firms' size (conditional on age) and as a function of age (conditional on size) respectively. We can conclude that in the stationary distribution all variables are decreasing in the size and in the age of the firm, even after controlling, respectively for age and size.

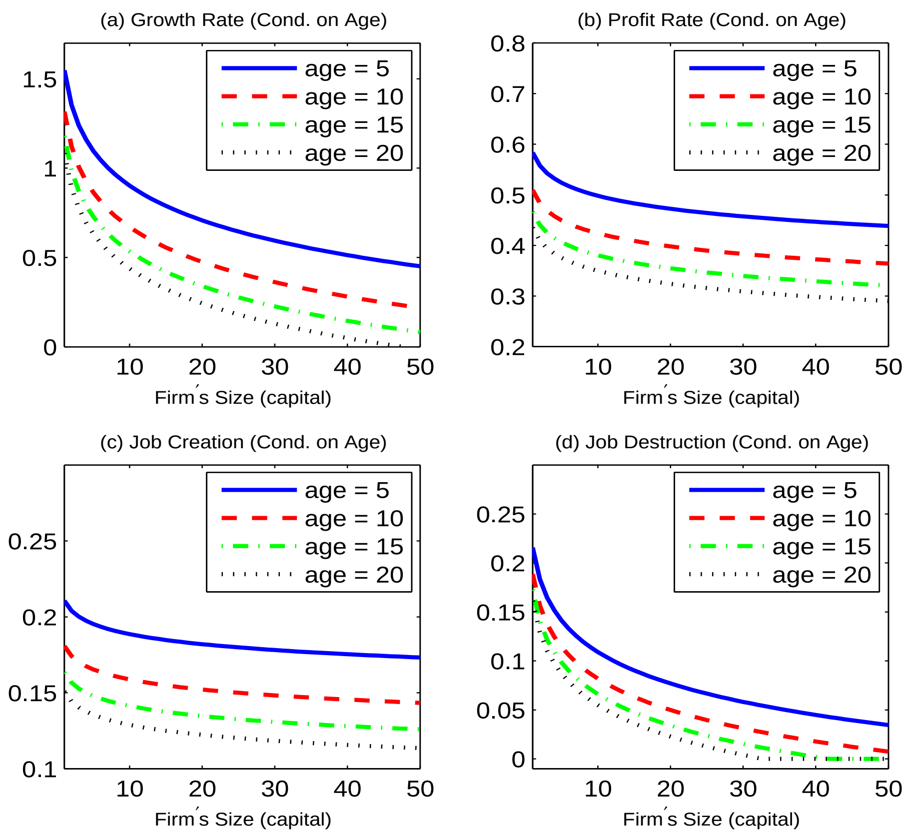


Figure 3.5: Size Dependence (Firms' Dynamics Conditional on Age).

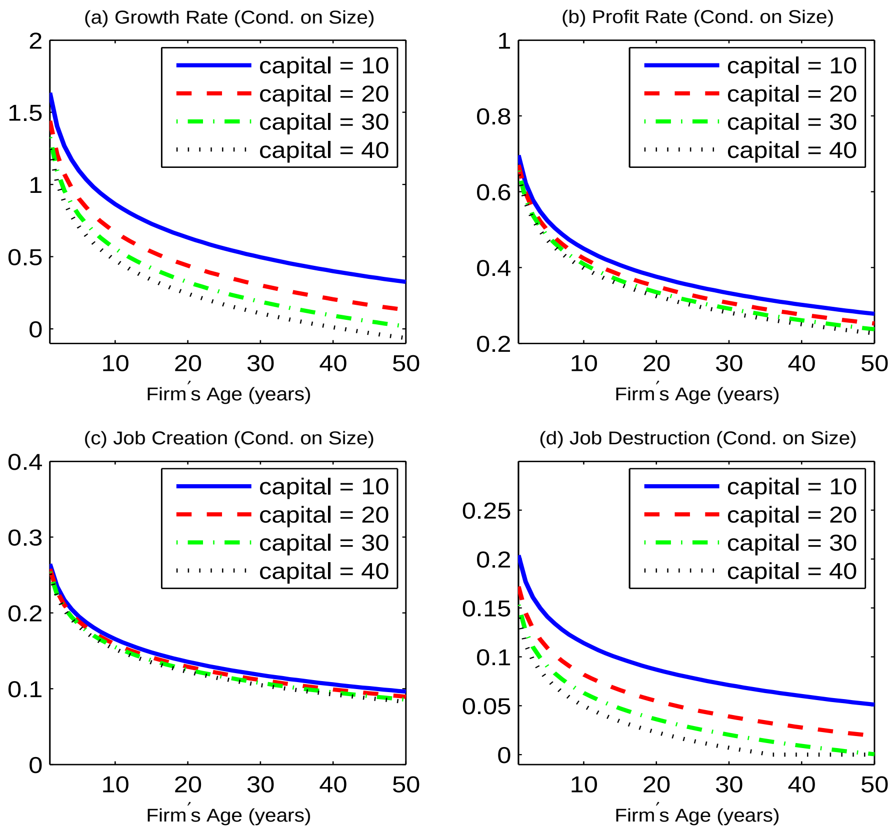


Figure 3.6: Age Dependence (Firms' Dynamics Conditional on Size).

The heterogeneity just described plays the most important role in generating the age dependence in the economy. This heterogeneity is derived from the distribution of shocks from where entrants draw their shock in the first period but more importantly from the persistence that idiosyncratic shocks display in the model. If the shocks present a high autocorrelation, the heterogenous composition is maintained for long periods, that is if ρ_a is close to one, the distribution of entrants $\nu(s)$ will shape the distribution of active firms for a long time. In the limit, if $\rho_a \to 1$ the distribution over shocks of active firms will be similar to $\nu(s)$. On the contrary, as $\rho_a \to 0$ only the distribution of very young firms will remain close to their initial distribution. To make it clearer consider the following example: assume that there are no disruption costs associated with adjusting the capital stock and $\pi_x = 0$. Then, the Euler equation of an active firm after substituting the envelope condition is

$$-1 - g_{k'}(k,k') + \frac{1}{(1+r)}\Big[\sum_{s'} \Pi(s'|s)R_{k'}(s',k') + (1-\delta k') + g_k(k',k'')\Big] = 0$$

$$\Rightarrow 1 + g_{k'}(k,k') = \frac{1}{(1+r)}\Big[\sum_{s'} \Pi(s'|s)R_{k'}(s',k') + (1-\delta k') + g_k(k',k'')\Big] \quad (3.19)$$

This is the usual capital accumulation equation of a firm where the left hand side represent the marginal costs and the left hand side represents the marginal benefits of investment. Consider the extreme case where shocks are iid and distributed according to the stationary distribution corresponding with Π. Denote this distribution with Π^*. In this world only the shocks of entrants will depend on $\nu(s)$. For any firm with age greater or equal to 2, the shocks will be drawn from Π^*. Then the capital accumulation equation becomes

$$1 + g_{k'}(k,k') = \frac{1}{(1+r)}\Big[\sum_{s'} \Pi^*(s')R_{k'}(s',k') + (1-\delta k') + g_k(k',k'')\Big] \quad (3.20)$$

that is independent of s. Thus, once you condition on size, the capital accumulation of the firm is independent of the shock and then firms with different ages will behave as identical firms conditional on the capital stock. The distribution of firms for active plants that are not entrants is the same across shocks.

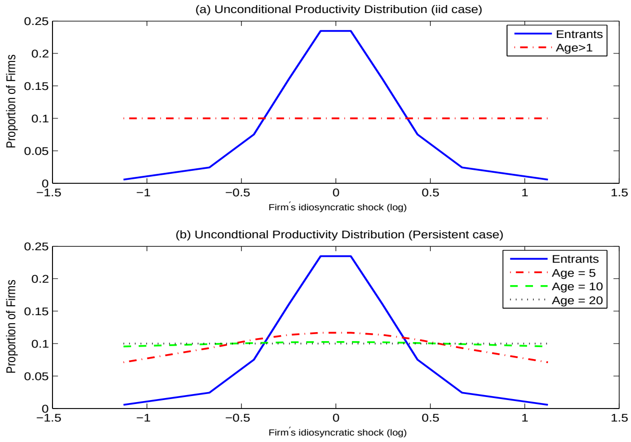


Figure 3.7: Distribution of firms over shocks for different ages

If you add some level of persistence, not only the decision of the firm will depend on the idiosyncratic shock, but also the distribution of firms across shocks will differ for firms with different ages. The optimal capital investment is the solution to problem (3.6). The measure of active firms with a particular shock will approach $\Pi^*(s)$ as you consider older firms. Figure 3.7 display the distribution over shocks for different age values. In the iid case the differences in technology composition disappears after the first period. In the persistent case, these differences are present for firms with more than 20 years of life.

In Figure 3.7 each line corresponds to a different age and in the iid case only two lines can be distinguished. After the first year of production, the distribution of firms over s is the same across firms with different ages. For the case where the shocks are persistent, the technological differences prevail for a long time. Cooley and Quadrini (2001) needed to assume that the entrants were of a particular technology type to generate the right sign in the age dependence (see Cooley and Quadrini (2001) page 1303). In our model, the size of the entrants as well as the entry barrier are calibrated to match the facts observed in the U.S. manufacturing sector.

3.7.1 Evidence on Financial Constraints

Cooley and Quadrini (2001) motivated the introduction of financial constraints in a standard model of firm dynamics pointing to the relation between investment rates and Tobin's Q and cash flows. However, there are theoretical arguments and empirical evidence showing that investment-cash flows sensitivities are not good indicators of financing constraints or financial frictions. Cooper and Ejarque (2001) find that the sensitivity of investment rates to cash flows does necessarily comes from a model with financial constraints. They estimate different models of capital investment to match the "Q-theory" regressions and obtain a better fit with a model with no financial frictions. Moreover, Erickson and Whited (2000) and Gomes (2001) argue that the relation between investment rates and Tobin's q comes from measurement error. In my model, the monotonicity of the investment function imply that investment of firms is sensitive to cash flows generating the significant relation obtain in the data. To show this I simulate my model economy and apply the same econometric procedures that previous studies pointing to financial constraints used. The estimated model takes the following form

$$\frac{i_{j,t}}{k_t} = a_0 + a_1 E[q_{j,t+1}] + a_2 \frac{R_{j,t}}{k_{j,t}} + \epsilon_t \tag{3.21}$$

where the subscript j denotes firm j and t corresponds to the time period. A significant coefficient a_2 in this type of regression motivated the inclusion of financial constraints. The results from my model are displayed in Table (3.4).

Coefficient	Value	Std. Error
Tobin's Q	0.087	1.92e-06
Profit Rate	2.95	5.71e-04

Table 3.4: Model Predictions.

The values obtained are in line with the estimated coefficients reported by Gomes (2001) and Cooper and Ejarque (2001). The goal of this exercise was to understand the "cash-flow effect" and the relation with financial constraints. We observe that a model with no financial frictions and only some level of capital irreversibility also generates a significant "cash flow" coefficient. This is not an argument against financial frictions. I do not question the existence or importance of these constraints for investment decisions. Nevertheless, this result cast serious doubt on the common interpretation of cash-flow effects as evidence in favor of financing constraints.

Hence, the integration of a basic model of industry dynamics with non-convex adjustment costs and entry and exit is able to capture most of the stylized facts about the investment behavior and the growth of firms. In particular, we are able to reproduce the conditional age and size dependence that the empirical literature pointed before and that previous models of investment and firms' dynamics were not able to obtain. In contrast with previous models were financial frictions were necessary to address this question we developed a model were the friction present is the adjustment costs of capital accumulation.

3.8 Conclusion

Models of firm and industry dynamics that consider entry and exit were unable to account simultaneously for the conditional dependence of firm growth, standard deviation of growth and job reallocation on size and age. Cooley and Quadrini (2001) point out that one possible explanation could be the introduction of financial frictions in an otherwise standard model. They show that the integration of persistent shocks and financial-market frictions allows the model to generate the desired firm dynamics. In this paper, I argued that a model of investment dynamics with adjustment costs that introduces entry and exit can also account for the conditional size and age dependence.

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